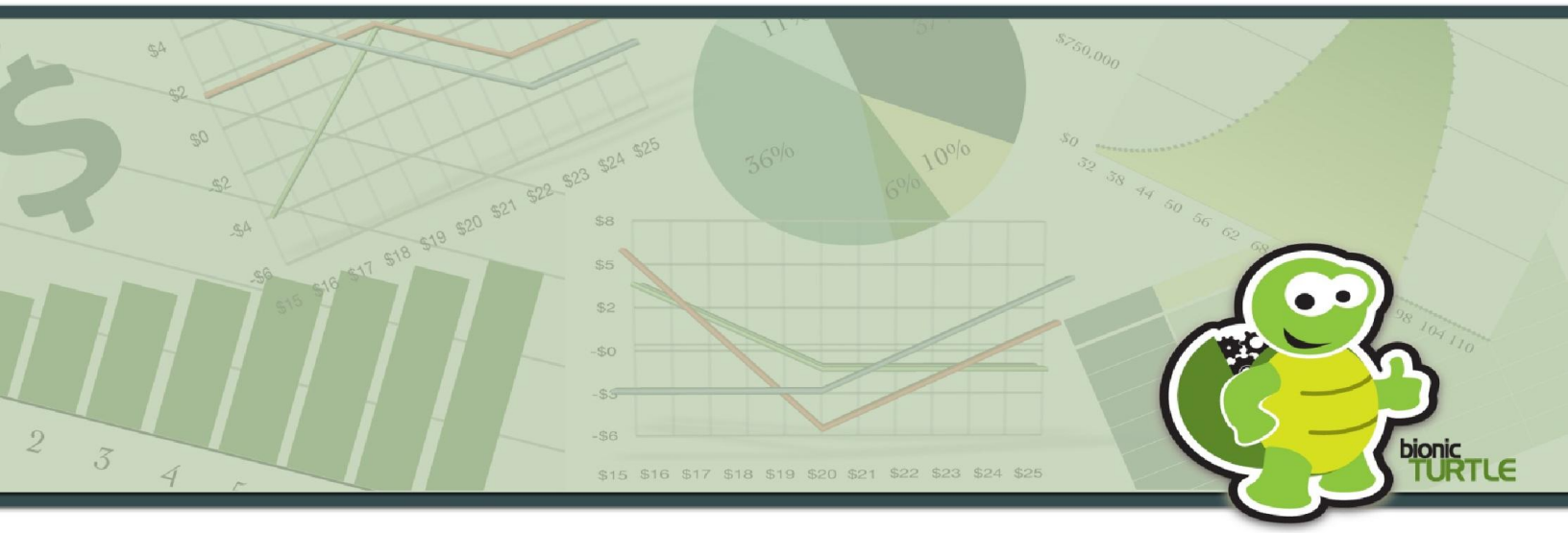

P1.T4. Valuation & Risk Models

Chapter 13. Modeling and Hedging Non-Parallel Term Structure Shifts

Bionic Turtle FRM Study Notes

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- **Describe and assess the major weakness attributable to single-factor approaches when hedging portfolios or implementing asset liability techniques.**
- **Describe the principal components analysis and explain its use in understanding term structure movements.**
- **Define key rate exposures and know the characteristics of key rate exposure factors including partial '01s and forward-bucket '01s.**
- **Describe key-rate shift analysis.**
- **Define, calculate, and interpret key rate '01 and key rate duration.**
- **Describe the key rate exposure technique in multi-factor hedging applications and summarize its advantages and disadvantages.**
- **Calculate the key rate exposures for a given security and compute the appropriate hedging positions given a specific key rate exposure profile.**
- **Relate key rates, partial '01s and forward-bucket '01s, and calculate the forward bucket '01 for a shift in rates in one or more buckets.**
- **Apply key rate and multi-factor analysis to estimating portfolio volatility**

Describe and assess the major weakness attributable to single-factor approaches when hedging portfolios or implementing asset liability techniques.

Yield-based duration, DV01, and convexity are *single-factor* approaches because their sole interest rate factor is yield-to-maturity (YTM). To simulate a shock to the yield is to implicitly assume a parallel shift in the term structure (and further, the influence on price is estimated only as a linear approximation)[1]. **The major weakness of a single-factor approach is that it will not address curve risk**. Curve risk is the risk that term structure dynamics include non-parallel shifts. According to Fabozzi[2], the two primary non-parallel shifts are:

- Twist: a flattening or steepening
- Butterfly: Change in curvature or "humpedenss"; i.e., a positive (negative) butterfly is when the curve becomes more (less) humped. We call also think of butterflies as changes in the degree of concavity or convexity.

This inability of yield-based approaches (as our proxy for the most common single-factor approach) to manage curve risk obviously manifests when hedging fixed-income portfolios: we can neutralize duration such that our net portfolio achieves a zero DV01, but the hedge is maintained only for small, parallel shifts in the term structure. If the term structure exhibits a twist or butterfly (or other exotic but realistic dynamic) our hedge will not perform.

[1] In the previous chapter we showed that yield-to-maturity (aka, yield) shocks approximate a parallel shift. A single-factor model does not necessarily imply a parallel shift—as we can imagine various functions of a single factor—however, yield-to-maturity approaches do effectively assume a parallel shift.

[2] Fabozzi, Frank. Fixed Income Analysis Wiley; 2 edition (January 22, 2007). Page 188.

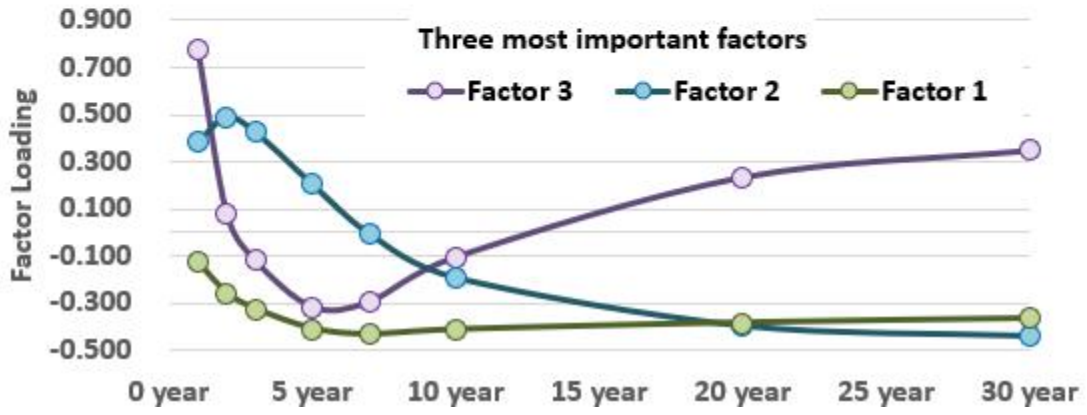
Describe the principal components analysis and explain its use in understanding term structure movements.

Principal component analysis (PCA)[3] is a statistical technique that achieves *dimension reduction*. Dimension reduction *simplifies* the complex dynamics of an interest rate term structure: while rates are realistically determined by many factors, PCA reduces to a handful the number of factors that *predominantly* determine rate dynamics. As Carol Alexander explains,

> "PCA is the simplest of many orthogonalization techniques that transform a set of correlated variables into a set of uncorrelated variables … PCA can be applied to any set of stationary time series, however high or low their correlation, but it works best on highly correlated systems such as a set of zero coupon returns of different maturities, a set of commodity futures returns of different maturities, or an implied volatility surface. In short, PCA works best for term structures."
> —Carol Alexander, Market Risk Analysis (Volume I)[4]

GARP's example[5] illustrates an analysis (PCA) performed on eight daily Treasury rates and eight factors. The values inside the matrix (e.g., -0.129) are called *factor loadings*.

Rate	Factors							
Maturity	1	2	3	4	5	6	7	8
1 year	-0.129	0.384	0.778	-0.479	-0.009	0.003	-0.017	-0.007
2 year	-0.258	0.490	0.081	0.538	0.588	0.000	0.025	0.001
3 year	-0.326	0.426	-0.119	0.241	-0.727	-0.292	0.150	0.062
5 year	-0.410	0.203	-0.317	-0.216	-0.068	0.574	-0.548	-0.096
7 year	-0.434	-0.008	-0.294	-0.397	0.221	0.099	0.705	0.110
10 year	-0.414	-0.193	-0.104	-0.204	0.211	-0.707	-0.338	-0.284
20 year	-0.385	-0.397	0.234	0.161	-0.023	0.019	-0.169	0.764
30 year	-0.367	-0.440	0.348	0.317	-0.164	0.275	0.193	-0.557
Std Dev	14.150	4.910	2.440	1.590	1.090	0.850	0.780	0.680



[3] Tuckman, Bruce and Angel Serrat. Fixed Income Securities: Tools for Today's Markets. Wiley 2012. Formulas 4.6
[4] Alexander, Carol, Market Risk Analysis, Quantitative Methods in Finance. Wiley; Volume I edition (May 27, 2008)
[5] 2020 FRM Part I: Valuation and Risk Models, 10th Edition [VitalSource Bookshelf version]. Section 13.1. Spreadsheets are hand built by David Harper

The values (aka, factor loadings) are analogous to betas or regression coefficients: if there is a +1.0 change in Factor 3, then the 1-year rate moves up by **0.778 basis points (see purple column in prior exhibit)**, the 2-year rate moves up by **0.081 basis points**, and the 3-year rate moves down by **0.119 basis points**.

The change in the j-th rate (e.g., 2-year Treasury rate) on a given day is given by the following[6] *linear combination* where a(i) is the *factor score* and f(i,j) is the *factor loading* for the i-th factor and the j-the rate.

$$\sum_{i=1}^{8} a_i f_{ij}$$

Because the factors (by definition of the PCA analysis. Yes, PCA analysis is redundant!) are orthogonal, the total variance is the sum of the variance of the factor scores. For GARP's data, the total variance is: $14.15^2 + 4.91^2 + \ldots + 0.68^2 = 235.8$. See the exhibit below.

Standard Deviation of Factor Scores								Tot	
1	2	3	4	5	6	7	8	Variance	Explains
14.150	**4.910**	**2.440**	**1.590**	**1.090**	**0.850**	**0.780**	**0.680**		
200.22								200.2	**84.9%**
200.22	24.11							224.3	**95.1%**
200.22	24.11	5.95						230.3	**97.7%**
200.22	24.11	5.95	2.53					232.8	**98.7%**
200.22	24.11	5.95	2.53	1.19				234.0	**99.2%**
200.22	24.11	5.95	2.53	1.19	0.72			234.7	**99.5%**
200.22	24.11	5.95	2.53	1.19	0.72	0.61		235.3	**99.8%**
200.22	24.11	5.95	2.53	1.19	0.72	0.61	0.46	235.8	**100.0%**

The first factor explains 84.9% ($=14.15^2/235.77$) of the total variance. The first two factors explain

$$\frac{14.15^2 + 4.91^2}{235.77} = 95.1\%$$

of the total variance. This PCA analysis is typical in the sense that the first three factors explain over 90.0% of the total variance; in this case, the first three factors explain 97.7% of the variance. We might not need to include more than three or four factors.

PCA is a statistical technique that does not presume a fundamental narrative. There is nothing natural or necessary to suggest that the first factor reflects a parallel shift. However, we can *observe the pattern of factor loadings* (as plotted on the previous change). In this case, we can see visually that the top three factors do exhibit patterns.

[6] 2020 FRM Part I: Valuation and Risk Models, 10th Edition [VitalSource Bookshelf version]. Section 13.1. Spreadsheet hand built by David Harper