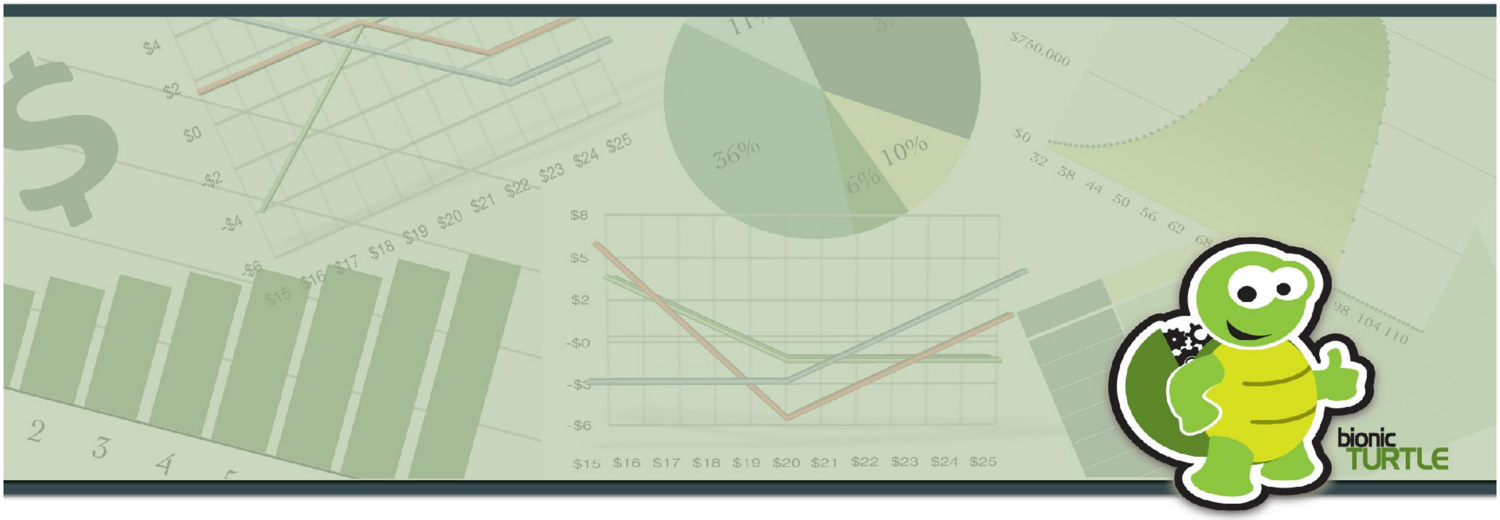

P1.T4. Valuation & Risk Models

Chapter 2. Calculating and Applying VaR

Bionic Turtle FRM Study Notes

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- **Explain and give examples of linear and non-linear derivatives.**
- **Describe and calculate VaR for linear derivatives.**
- **Describe and explain the historical simulation approach for computing VaR and ES.**
- **Describe the delta-normal approach to calculating VaR for non-linear derivatives.**
- **Describe the limitations of the delta-normal method.**
- **Explain the full revaluation method for computing VaR.**
- **Compare delta-normal and full revaluation approaches for computing VaR.**
- **Explain structural Monte Carlo, stress testing methods for computing VaR, identifying strengths and weaknesses of each approach.**
- **Describe the implications of correlation breakdown for scenario analysis.**

Selected key concepts:

- The most general form of normal linear (parametric) value at risk (VaR) is given by

  $$\mathbf{aVaR_{P/L}} = -\left(\frac{\Delta t}{T}\right)\boldsymbol{\mu_{P/L}} + \sqrt{\frac{\Delta t}{T}} \cdot \boldsymbol{\sigma_{P/L}} \cdot \mathbf{z_{\alpha}} \ .$$

  The FRM candidate must be comfortable with incorporating the drift, μ, and scaling the VaR over time (including scaling the volatility per the square root rule). This *absolute VaR* is the worst expected loss relative to the initial (today's) position; if we omit the drift term, then we are retrieving the *relative VaR* which is relative to the position's expected future value. Often these terms (absolute versus relative) are not explicitly written, but we should look for the context to ascertain whether we want to identify the worst expected loss relative to either today's *current* value, or relative to the expected *future* value. For a short horizon—notably, the one-day VaR—the expected return is often assumed to be zero (i.e., μ = 0) such that the absolute/relative distinction is irrelevant. But it matters for longer horizons.

- In the delta-normal approach (which is a parametric or analytical approach), the linear approximation is assumed (i.e., as if the derivative were linear) and the underlying factor is assumed to follow a normal distribution.

- Historical simulation (HS) sorts the portfolio's losses (either as directly observed in the window; or by applying historical risk factor shocks/changes to the current portfolio) and then retrieves the risk measure (e.g., VaR, ES) directly from the sorted losses because they comprise an empirical distribution. Unlike delta-normal VaR which is parametric, HS is non-parametric. The chief advantage of HS is that neither a distributional assumption nor distributional parameters are needed: heavy tails, or any other "messy" features, can be a natural feature of the data.

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On the calculation of normal linear value at risk (VaR)

P/L versus L/P format: The *native* numerical format is signified by **P(+)/L(-)** or **P/L**. This is native because profits (aka, gains) are positive and losses are negatives. The worst expected loss (aka, VaR) is surely a negative in P/L format; e.g., if our P/L distribution happens to be conveniently characterized by a random standard normal variable, Z ~ N(0, 1), then in P/L format a profit might be +0.30 but the 95.0% VaR is -1.645. However, in risk we tend to work in the loss tail, so we often utilize **L(+)/P(-)** or **L/P** format. In L/P format, losses are positives and therefore gains (or profits) are negatives. Assuming the exact same random Z, the 95.0% VaR in L/P format is +1.645 because in L/P the right-hand side of the distribution contains the losses. Equity positions are expected to gain over time (aka, drift positively); in L/P format, an equity portfolio with positive expected return of +9.0% would be represented in L/P format by a negative, μ = -9.0% because the left-side of the L/P distribution contains the gains in L/P format.

Absolute VaR: Imagine a position with a current value, W(0) = $100.00, that has an expected return (aka, positive drift) of 9.0% per annum with volatility (aka, standard deviation) 20.0% per annum. See the exhibit below. We'll assume 250 trading days per year. In P/L format, we have μ = +0.090 and σ = 0.20. The **general P/L form** of VaR is given by the following absolute VaR (left) and corresponding relative VaR (right):

$$\mathbf{aVaR_{P/L}} = -\left(\frac{\Delta t}{T}\right)\mu_{P/L} + \sqrt{\frac{\Delta t}{T}} \cdot \sigma_{P/L} \cdot z_\alpha \rightarrow \mathbf{rVaR_{P/L}} = \sqrt{\frac{\Delta t}{T}} \cdot \sigma_{P/L} \cdot z_\alpha$$

Let's say we seek the normal 95.0% 10-day VaR of this position; ΔT = 10 days. The **absolute VaR** equals -(10/250) × 0.090 + sqrt(10/250) × 0.20 × 1.645 = 0.0622 or 6.22%; in dollar terms, 6.22% × $100.0 = $6.22. The corresponding **relative VaR,** which omits the drift term, is sqrt(10/250) × 0.20 × 1.645 = 6.58%, or $6.58 in dollar terms. Absolute VaR is the worst expected loss *relative to the initial (current) position*, while relative VaR is the worst expected loss *relative to the position's expected future value*. As we might expect, here, the difference is the drift of (10/250) × 0.090 = 0.36% or $0.36. Finally, if we are using instead the L/P format, then we must get the same result! Consequently, the L/P version is given by:

$$\mathbf{aVaR_{L/P}} = \left(\frac{\Delta t}{T}\right)\mu_{L/P} + \sqrt{\frac{\Delta t}{T}} \cdot \sigma_{L/P} \cdot z_\alpha \rightarrow \mathbf{rVaR_{L/P}} = \sqrt{\frac{\Delta t}{T}} \cdot \sigma_{L/P} \cdot z_\alpha$$

Trading days per year	250	
VaR horizon (days)	10	
VaR confidence level, c	95.0%	→ 1.645
Asset Value ($)	$100.0	
Asset Expected return, per annum	9.0%	
Volatility, per annum	20.0%	

VaR	Absolute	Relative
Value at Risk (VaR), per annum, %	23.9%	32.9%
Value at Risk (VaR), per annum, $	$23.90	$32.90
Value at Risk (VaR), horizon, %	6.22%	6.58%
Value at Risk (VaR), horizon, $	$6.22	$6.58

Explain and give examples of linear and non-linear derivatives.

A linear derivative has a linear relationship between the derivative's price and the derivative's underlying risk factor (aka, pricing factor). "It does not need to be one-for-one but the transmission parameter or delta (sensitivity of derivatives price to changes in the underlying factor's price) needs to be constant for all levels of the underlying factor," explains Linda Allen[1]. Therefore, a non-linear derivative has a *non-constant* delta.

- **Linear derivative:** Price of derivative is a *linear function of underlying asset*. The transmission parameter (aka, delta) is constant and not a function of the level of the underlying factor. For example, a futures contract on S&P 500 index is approximately linear. A single point change in the index translates into a change (increase or decrease) of $250 on the futures contract, irrespective of the index's level.
- **Non-linear derivative.** Price of derivative is a *non-linear function of underlying asset*. Here the delta changes with—or is a function of—the underlying factor. For example, a stock option is non-linear. Say when the underlying stock price is $100.00, the price of an at-the-money call option (ie, S = K = $100.00) is $12.22 and the option's delta is 0.61. This delta suggests that if the stock price increases by $1.00, the option's price will increase by about $0.61. If the stock jumps up to $110.00, the option's delta will shift to about 0.74. The option is non-linear because delta varies with the stock price, which is also a risk factor (along with volatility and the interest rate, eg).

Note: Even non-linear assets are *effectively **locally*** linear. For example, a stock option is a decidedly non-linear derivative: the option is convex in the value of the underlying. Put another way, an option's delta is not constant[2]. But for tiny (infinitesimal) asset price changes, the delta is approximately constant. So can say that delta is a linear *approximation* as the relationship is locally linear, although it breaks down for non-tiny price changes.

Describe and calculate VaR for linear derivatives.

In the case of a linear derivative, the transmission parameter is constant. Therefore, in the case of a linear derivative, VaR scales directly with the underlying risk factor.

$$\mathbf{VaR_{Linear\ Derivative}} = \Delta \times \mathbf{VaR_{Underlying\ Risk\ Factor}}$$

For example: $$\mathbf{VaR_{S\&P\ 500\ Futures\ Contract}} = \$\mathbf{250} \times \mathbf{VaR_{Index}}$$

Consider this typical FRM question that applies the idea.

2012 Practice Exam Part 1, Question 1: You have been asked to estimate the VaR of an investment in Big Pharma Inc. The company's stock is trading at USD $23.00 and the stock has a daily volatility of 1.5%. Using the delta-normal method, the VaR at the 95% confidence level of a long position in an at-the-money put on this stock with a delta of -0.5 over a 1-day holding period is closest to which of the following choices?

Answer: VaR = Δ * 1.645 * σ * S(0) = |-0.5| * 1.645 * 0.015 * 23 = 0.28.

Two notes about this question: First, notice how it is satisfied to apply the linear approximation using only option delta; it is **not a delta-gamma** approximation. Second, we must realize this is an *approximation*: the actual relationship is non-linear. We are omitting the curvature (option gamma).

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[1] Allen, Linda, Understanding Market, Credit, and Operational Risk (Blackwell Publishing Ltd 2004)

[2] Put even another way, an option is a non-linear because its *gamma* is nonzero; i.e., its delta is not constant.