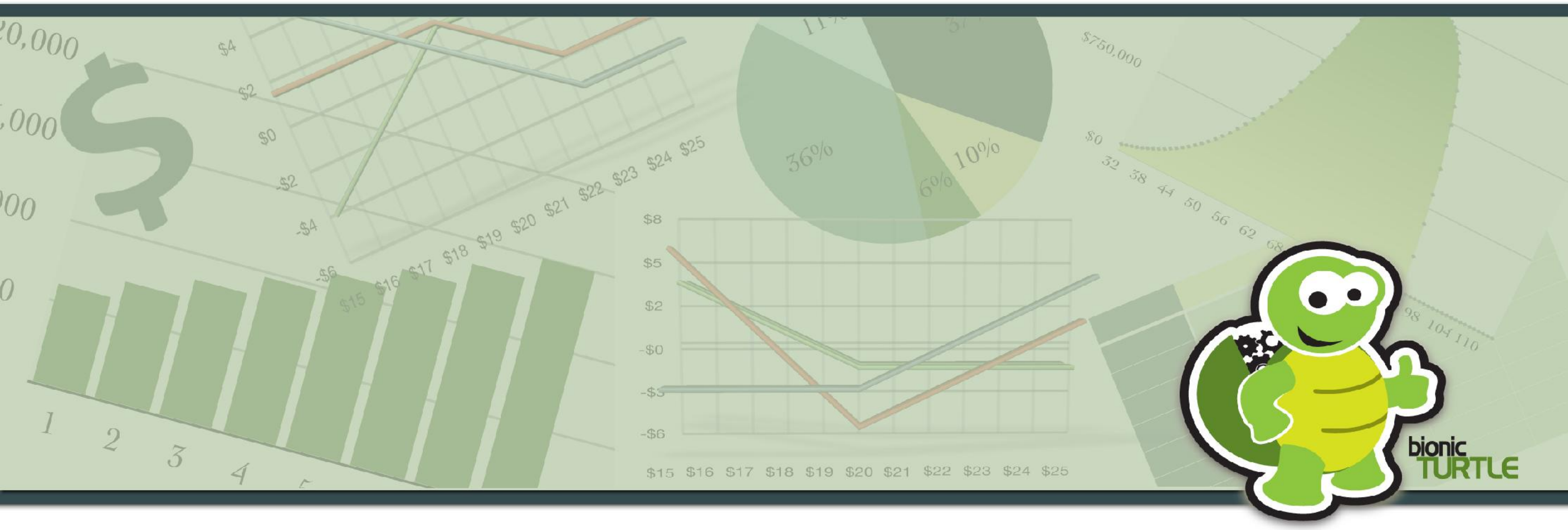


P2.T5. Market Risk Measurement & Management

Kevin Dowd, Measuring Market Risk

Bionic Turtle FRM Study Notes

Dowd Chapter 3: Estimating Market Risk Measures

Dowd, Chapter 4: Non Parametric Approaches

Dowd, Chapter 7: Parametric Approaches (II): Extreme Value

Dowd Chapter 3: Estimating Market Risk Measures

- **Estimate VaR using a historical simulation approach.**
- **Estimate VaR using a parametric approach for both normal and lognormal return distributions.**
- **Estimate the expected shortfall given P/L or return data.**
- **Define coherent risk measures.**
- **Estimate risk measures by estimating quantiles.**
- **Evaluate estimators of risk measures by estimating their standard errors.**
- **Interpret QQ plots to identify the characteristics of a distribution.**

Estimate VaR using a historical simulation approach.

The most common approach to the estimation of value at risk (VaR) is **basic historical simulation (HS).** This simple approach merely requires *sorting* the losses over some selected historical window. There are two steps:

1. Order (sort) the daily profit/loss observations.
2. Locate the loss corresponding to the specified confidence level; e.g., 95.0%, 99.0%

More generally, if we have (n) observations, and our confidence level is α, we would want the $[(1-\alpha) * n + 1]^{th}$ highest observation.

For example, if we have (n) observations, according to Dowd[1], the 95.0% VaR is the $(0.050 * n + 1)^{th}$ highest observation.

- Assume we have n = 1,000 loss observations and we want the 95.0% confident VaR. Because the confidence level implies a 5% tail, we know there are 50 observations in the tail, and we can assume the VaR to be the 51^{st} worst loss observation.
- Assume we have n = 500 loss observations (i.e., two years, each year with 250 trading days) and we want the 99.0% confident VaR. Because the confidence level implies a 1.0% tail, we know there are 5 observations in the tail, and we can assume the VaR to be the 6^{th} worst loss observation.

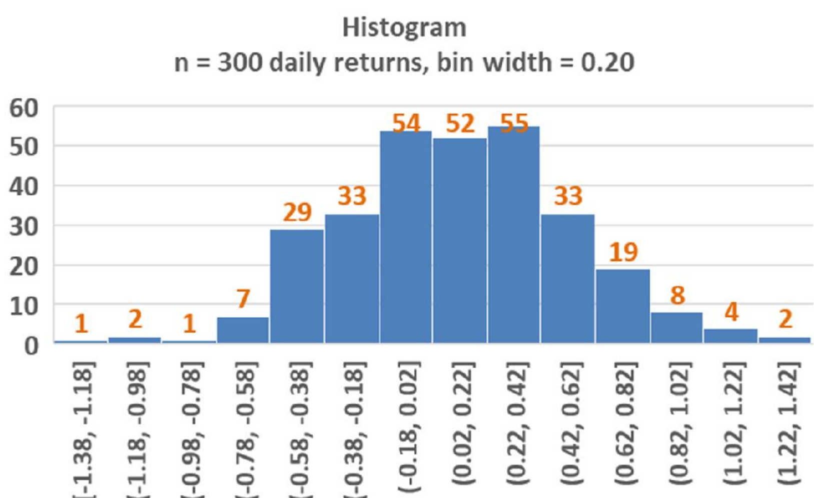
[1] Kevin Dowd, Measuring Market Risk, 2nd Edition (West Sussex, England: John Wiley & Sons, 2005)

Example: Assuming we have 300 daily P/L observations, on sorting the observations, the VaR corresponding to the 99.0% confidence level is the 300 * 1.0% + 1 = 4^{th} worst loss = $0.79.

If n = 300, then 99.0% HS VaR is the 4th worst loss			
1	-$1.380	11	-$0.590
2	-$1.160	12	-$0.570
3	-$0.990	13	-$0.550
4	-$0.790	14	-$0.550
5	-$0.710	15	-$0.540
6	-$0.700	16	-$0.540
7	-$0.700	17	-$0.530
8	-$0.670	18	-$0.530
9	-$0.650	19	-$0.520
10	-$0.590	20	-$0.510

Note that VaR is a loss but expressed as a positive typically. When losses (which are mathematically negatives of course) are rendered as positives, this format is referred to as L/P to signify Loss(+)/profit(-) rather than the more natural Profit(+)/loss(-) format.

The figure below shows the histogram of 300 hypothetical loss observations and the 99.0% VaR is 0.79. In practice, it is often helpful to obtain HS VaR estimates from a cumulative histogram, or empirical cumulative frequency function.



Estimate VaR using a parametric approach for both normal and lognormal return distributions.

In approaching the estimation of VaR, there is a fundamental difference between the simulation-based approaches (e.g., basic historical simulation) and parametric approaches. Historical simulation (HS) does not require, nor even desire, the specification of a statistical (aka, parametric) distribution; consequently, it does require a large dataset. On the other hand, parametric approaches assume a statistical distribution such as the normal or lognormal distributions. As an introduction, the two most common parametric distributions are the normal and the lognormal.

Normal value at risk (VaR)

Under the assumption that profit/loss is normally distributed, the VaR at confidence level alpha (α; please note Dowd[2] uses alpha to denote confidence whereas elsewhere we typically use alpha to denote significance!) is given by:

$$\alpha VaR = -\mu_{P/L} + \sigma_{P/L} z_{\alpha}$$

For example, given a mean of 10% and volatility of 20%, the **95% normal (relative) VaR** is 22.90 calculated as: -10% + 20% * 1.645.

Mean	10.0%
Standard Deviation	20.0%
Confidence Level (CL)	95.0%
Normal deviate	1.645
95% VaR	22.90%

Lognormal value at risk (VaR)

The lognormal VaR is given by: $\alpha VaR = P_{t-1}(1 - exp[\mu_R - \sigma_R z_{\alpha}])$

For example, assuming a mean of 10% and volatility of 20%, the **95% lognormal VaR** is 20.46, calculated as: 1 – exp [10% - (20% * 1.645)]

Mean	10.0%
Standard Deviation	20.0%
Confidence Level (CL)	95.0%
Normal deviate	1.645
95% VaR	20.46%

[2] Kevin Dowd, Measuring Market Risk, 2nd Edition (West Sussex, England: John Wiley & Sons, 2005)

Example: GARP's 2017 Practice Question #2 - A risk manager is estimating the market risk of the portfolio using both the normal distribution and lognormal distribution assumptions. He gathers the following data on the portfolio: Annual mean = 15.0%, Annual volatility = 35.0%, Current portfolio value = EUR 4,800,000, Trading days in a year = 252

Which of the following statement is correct?

A. Lognormal 95% VaR is less than normal 95% VaR at 1-day holding period by 0.13%
B. Lognormal 95% VaR is less than normal 95% VaR at 1-year holding period by 7.91%
C. Lognormal 99% VaR is less than normal 99% VaR at 1-day holding period by 1.43%
D. Lognormal 99% VaR is less than normal 99% VaR at 1-year holding period by 13.86 %

Solution: The correct answer is B as 42.570% - 34.669% = 7.901%

Liquidity-adjusted VaR		
Confidence level		**95.0%**
→ deviate, α =		**1.645**
Price per share, P		**$48.00**
Number of shares, N		**100,000**
Wealth, W = P*N		**$4,800,000**
	Annual	**Daily**
Exp return, μ	**15.00%**	**0.05952%**
Volatility, σ(daily)	**35.00%**	**2.20479%**
Value at Risk (VaR) Compared		
Absolute VaR (%), normal	**42.570%**	**3.567%**
Absolute VaR ($), normal	$2,043,354	$171,218
Absolute VaR (%), log	**34.669%**	**3.504%**
Absolute VaR ($), log	$1,664,097	$168,200
Difference	**7.901%**	**0.063%**

From the table on the previous page:

- The daily return is annual return divided by 252 = 15% / 252 = 0.05952%
- The daily volatility is annual volatility divided by sqrt (252) = 35% /sqrt (252) = 2.20479%

In percentage terms:

- 1 day normal 95% VaR is $-\mu_{P/L} + \sigma_{P/L} z_{\alpha}$ = -0.0592% + 2.20479% * 1.645 = 3.57%.
- 1 day lognormal 95% VaR is $(1 - exp[\mu_R - \sigma_R z_{\alpha}]$ = (1 - exp [0.05952% - 2.20479% * 1.645]) = 3.51%.
- So, the difference between normal and lognormal 95% VaR at the one year holding period is 0.063%.
- To arrive at the dollar value of VaR as shown in the table, the percentage VaR is multiplied by the portfolio value of $ 4.8 million.
- In a similar way, 1 year normal and lognormal 95% VaR is calculated as 42.570% and 34.669% respectively and their difference is 7.901%.
- Like this, the 1 day and 1 year normal and lognormal 99% VaR can be calculated (not shown here) and their differences found.

So, B is the right choice as the 95% lognormal VaR is lower than the 95% normal VaR at the one year holding period by 7.901%.