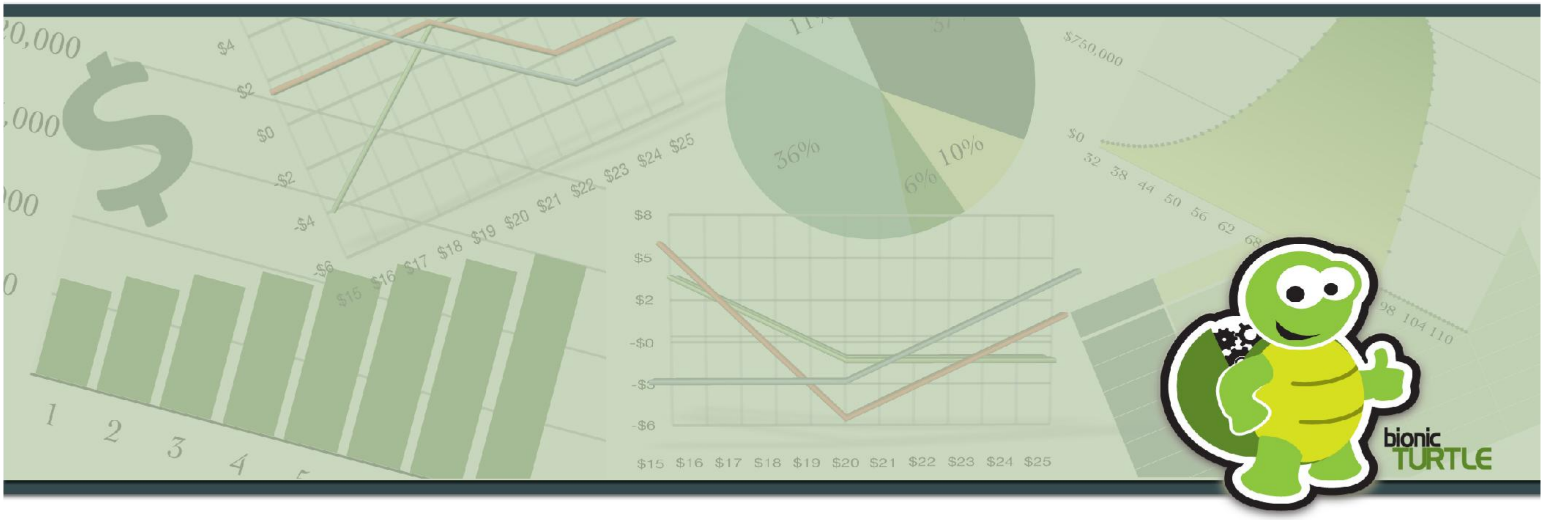


# P2.T5. Market Risk Measurement & Management

# Jorion, Value-at-Risk: The New Benchmark for Managing Financial Risk

# Bionic Turtle FRM Study Notes

**Jorion, Chapter 6: Backtesting VaR**



**Jorion, Chapter 11: VaR Mapping**

## Jorion, Chapter 6: Backtesting VaR

- **Define backtesting and exceptions and explain the importance of backtesting VaR models.**
- **Explain the significant difficulties in backtesting a VaR model.**
- **Verify a model based on exceptions or failure rates.**
- **Define and identify type I and type II errors.**
- **Explain the need to consider conditional coverage in the backtesting framework.**
- **Describe the Basel rules for backtesting.**

**Selected key concepts:**

- Key concepts to be included in next revision

## Define backtesting and exceptions and explain the importance of backtesting VaR models.

Model validation is the process of asking, **"Is this an adequate model?"** and/or **"Is this model consistent with reality?".** Validation tools include:

- Backtesting
- Stress testing
- Independent review and oversight

**Backtesting**: Backtesting attempts to verify whether *actual losses* are reasonably consistent with *projected losses*. It compares the history of value at risk (VaR) forecasts to actual (realized) portfolio returns. It is important to backtest VaR models because:

- Backtesting is a *reality check* on whether VaR forecasts are well-calibrated,
- Backtesting is key to Basel's decision to allow internal VaR models for regulatory capital requirements, and
- Under the Basel II internal models approach (IMA) to Market Risk, banks must back-test their VaR model; i.e., the green/yellow/red "traffic light" zones

**Exceptions:** We can easily backtest a VAR model if we accept the binomial distribution's assumptions (specifically, the iid assumption). When the VaR model is perfectly calibrated, the number of observations falling outside VAR should align with the confidence level. Specifically, **the percentage of observed exceptions should be approximately the same as the VaR significance level, where significance is one minus the confidence level**. The number of *exceptions* is also known as the number of *exceedances*, and this is simply the number of days during which the VaR level is exceeded.

- When **too many** exceptions are observed, the model is "bad" and underestimates risk. This is a major problem because too little capital may be allocated to risk-taking units; penalties also may be imposed by the regulator.
- When **too few** exceptions are observed, this also problematic because it leads to an inefficient allocation of capital across units.
- A **good (aka, accurate) model** will produce approximately the number of expected exceptions. For example, over 250 days, a good (aka, accurate) 95.0% VaR model will produce approximately 5.0% * 500 days = 25 exceptions. Over 100 days, a good 99.0% VaR model is expected to produce only 1.0% * 100 = 1 exception.

In summary, the number of loss observations (e.g., daily losses) that exceed the VaR is called the number of exceedances or exceptions. For example, if the VaR model is perfectly calibrated:

- A 95.0% daily VaR should be exceeded about 13 days per year; 5% * 250 days = 12.5 days (or 12.6 days if 252 days per year is assumed)
- A 99.0% daily VaR should be exceeded about 8 days per three years; 3 years* 250 days/year * 1.0% = 7.5 days (or 7.6 days if 252 days per year is assumed)

***Philippe Jorion writes this about backtesting*:** "Backtesting is a formal statistical framework that consists of verifying that actual losses are in line with projected losses. This involves systematically comparing the history of VAR forecasts with their associated portfolio returns.

These procedures, sometimes called reality checks, are essential for VAR users and risk managers, who need to check that their VAR forecasts are well calibrated. If not, the models should be reexamined for faulty assumptions, wrong parameters, or inaccurate modeling. This process also provides ideas for improvement and as a result should be an integral part of all VAR systems.

Backtesting is also central to the Basel Committee's ground-breaking decision to allow internal VAR models for capital requirements. It is unlikely the Basel Committee would have done so without the discipline of a rigorous backtesting mechanism. Otherwise, banks may have an incentive to understate their risk. This is why the backtesting framework should be designed to maximize the probability of catching banks that willfully understate their risk. On the other hand, the system also should avoid unduly penalizing banks whose VAR is exceeded simply because of bad luck. This delicate choice is at the heart of statistical decision procedures for backtesting." – Philippe Jorion [1]

## Explain the significant difficulties in backtesting a VaR model.

There are at least two difficulties when backtesting a VaR model:

- Backtesting remains a statistical decision to accept or reject (a null hypothesis) such that we cannot avoid the risk of committing one of the two possible errors (i.e., **Type I and Type II error**). Consequently, backtesting can never tell us *ex-ante* with 100.0% confidence whether our model is good or bad. Our decision to deem the model good or bad is itself a probabilistic (less than certain) evaluation.
- An actual portfolio is "contaminated" by (dynamic) changes in portfolio composition (i.e., trades and fees), but the VaR assumes a static portfolio.
    - Contamination will be minimized only in short horizons
    - Risk managers should track both the actual portfolio return and the hypothetical return (representing a static portfolio)
        - If the model passes backtesting with hypothetical but not actual returns, then the problem lies with intraday trading.
        - In contrast, if the model does not pass backtesting with hypothetical returns, then the modeling methodology should be reexamined
    - Sometimes a *cleaned-return* approximation is used instead of actual return which is actual return minus all non-mark-to-market items like fees, commissions, and net income.

[1] Jorion, Philippe, Value at Risk: The New Benchmark for Managing Financial Risk (McGraw-Hill Education; 3rd edition, November 9, 2006)

## Verify a model based on exceptions or failure rates.

We verify a model by recording the ***failure rate*** which is the proportion of times VaR is exceeded in a given sample. Under the *null hypothesis* of a correctly calibrated model (Null $H_0$: correct model), the number of exceptions (x) follows a binomial probability distribution:

$$f(x) = \binom{T}{x} p^X (1-p)^{T-x}$$

The expected value of (x) is p*T and a variance, σ^2(x) = p*(1-p)*T. By characterizing failures with a binomial distribution, we are assuming that exceptions (failures) are **independent and identically distributed (i.i.d.)** random variables.

**Let's illustrate with Jorion's own example**[2]. The assumptions are:

- The backtest (aka, estimation) window is one year with 250 trading days; T = 250
- The bank employed a 99.0% confidence value at risk (VaR) model; p = 0.010

The backtest analyzes the results of an actual, observed (realized) series of results. **Because each daily outcome either exceeded the VaR or did not, the historical window of observations is characterized by an (iid) binomial distribution**.

The table below illustrates (*on the left*) the distribution **if the model is calibrated correctly** when p = 1.0%. Note that for a given significance level (here p = 0.010 which corresponds to a 99.0% Var model), there can be only one correct binomial model. On the *right-hand* side of the exhibit, we illustrate **four different incorrect models**; i.e., p = 2.0%, 3.0% or 4.0%.

| 99.0% VaR Correct Model | |
|---|---|
| No. of Except | p= 0.01 T=250 |
| 0 | 8.11% |
| 1 | 20.5% |
| 2 | 25.7% |
| 3 | 21.5% |
| 4 | 13.4% |
| 5 | 6.7% |
| 6 | 2.7% |
| 7 | 1.0% |
| 8 | 0.3% |
| 9 | 0.1% |
| 10 | 0.0% |

| We believe our model is a 99.0% VaR model, but in truth, it is an Incorrect model | | | | |
|---|---|---|---|---|
| No. of Except | p= 0.02 T=250 | p= 0.03 T=250 | p= 0.04 T=250 | p= 0.05 T=250 |
| 0 | 0.6% | 0.0% | 0.0% | 0.0% |
| 1 | 3.3% | 0.4% | 0.0% | 0.0% |
| 2 | 8.3% | 1.5% | 0.2% | 0.0% |
| 3 | 14.0% | 3.8% | 0.7% | 0.1% |
| 4 | 17.7% | 7.2% | 1.8% | 0.3% |
| 5 | 17.7% | 10.9% | 3.6% | 0.9% |
| 6 | 14.8% | 13.8% | 6.2% | 1.8% |
| 7 | 10.5% | 14.9% | 9.0% | 3.4% |
| 8 | 6.5% | 14.0% | 11.3% | 5.4% |
| 9 | 3.6% | 11.6% | 12.7% | 7.6% |
| 10 | 1.8% | 8.6% | 12.8% | 9.6% |
| 11 | 0.8% | 5.8% | 11.6% | 11.1% |
| 12 | 0.3% | 3.6% | 9.6% | 11.6% |
| 13 | 0.1% | 2.0% | 7.3% | 11.2% |
| 14 | 0.0% | 1.1% | 5.2% | 10.0% |
| 15 | 0.0% | 0.5% | 3.4% | 8.2% |

[2] Jorion, Philippe, Value at Risk: The New Benchmark for Managing Financial Risk (McGraw-Hill Education; 3rd edition, November 9, 2006). Numerical examples are Jorion's but the spreadsheet exhibits were hand built by David Harper