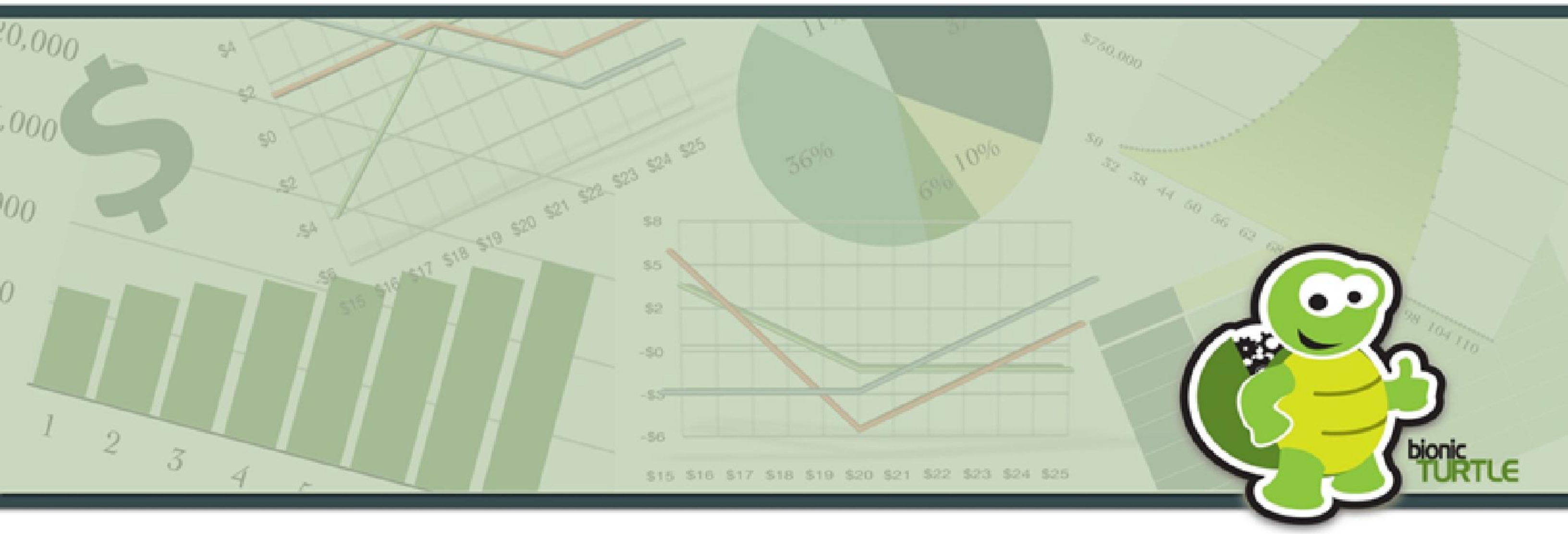

P1.T2. Quantitative Analysis

Bionic Turtle FRM Practice Questions

Chapter 1: Fundamentals of Probability

This is a super-collection of quantitative practice questions. It represents several years of cumulative history mapped to the current reading. Previous readings include Miller, Stock, and Gujarti, which we have retained in this practice question set.

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Note that this pertains to Chapters 1-6 in Topic 2, Quantitative Analysis. We will include this introduction in each of those practice question sets for reference.
Within each chapter, our practice questions are sequenced in *reverse chronological order* (appearing first are the questions written most recently). For example, consider Miller's Chapter 2 (Probabilities), you will notice there are fully three (3) sets of questions:

- Questions T2.708 to 709 (Miller Chapter 2) were written in 2017. The **7**XX denotes 2017.
- Questions T2.300 to 301 (Miller Chapter 2 were written in 2013. The **3**XX denotes 2103.
- Questions T2.201 to 207 (Stock & Watson) were written in 2012. Relevant but optional.

The reason we include the prior questions is simple: although the FRM's econometrics readings have churned in recent years (specifically, for Probabilities and Statistics, from Gujarati to Stock and Watson to Miller), the **learning objectives (AIMs) have remain essentially unchanged**. The testable concepts themselves, in this case, are generally quite durable over time. **Therefore, do not feel obligated to review all of the questions in this document!** Rather, consider the additional questions as merely a *supplemental, optional* resource for those who will to spend additional time with the concepts.

The major sections are:

- **This Chapter: Fundamentals of Probabilities (current QA-1, Chapter 1)**
  - Most Recent BT questions, Miller Chapter 2 (T2.708 & T2.709)
  - Previous BT questions, Miller Chapter 2 (T2.300 to T2.301)
  - Previous BT questions, Stock & Watson Chapter 2 (T2.201 to T2.207)

- **Random Variables (current QA-2, Chapter 2)**
  - Most Recent BT questions, Miller Chapter 3 (T2.710 to T2.712)
  - Previous BT questions, Miller Chapter 3 (T2.303 to T2.308)
  - Previous BT questions, Stock & Watson Chapter 3 (T2.208 to T2.213)
  - Previous BT questions, Gujarati (T2.57 to T2.82)

- **Common Univariate Random Variables (current QA-3, Chapter 3)**
  - Most Recent BT questions, Miller Chapter 4 (T2.713 to T2.716)
  - Previous BT questions, Miller Chapter 4 (T2.309 to T2.312)
  - Previous BT questions, Rachev Chapters 2 & 3 (T2.110 to T2.126)

- **Multivariate Random Variables (current QA-4, Chapter 4)**
  - Most Recent BT questions
    - Miller Ch.2 (T2.709)
    - Miller Ch.3 (T2.711)
    - Miller Ch.4 (T2.716)
  - Previous BT questions
    - Miller Ch.2 (T2.301)
    - Miller Chapter 3 (T2.304)
    - Stock & Watson Chapter 2 (T2.201 to T2.202)
    - Stock & Watson Chapter 3 (T2.212 to T2.213)
    - Gujarati (T2.57, T2.58, T2.62, T2.64, T2.65 & T2.67)

- **Sample Moments (current QA-5, Chapter 5)**
    - Most Recent BT questions, Miller Chapter 3 (T2.710 to T2.712)
    - Previous BT questions, Miller Chapter 3 (T2.303 to T2.308)
    - Previous BT questions, Stock & Watson Chapter 3 (T2.212 & T2.213)
    - Previous BT questions, Gujarati (T2.62 to T2.78)

- **Hypothesis Testing & Confidence Intervals (current QA-6, Chapter 6)**
    - Most Recent BT questions, Miller Chapter 7 (T2.718 & T2.719)
    - Previous BT questions, Miller Chapter 5 (T2.313 – T2.315)

- **Appendix**
    - Annotated Gujarati (encompassing, highly relevant)

Probabilities - Key Ideas

- Risk measurement is largely the quantification of uncertainty. We quantify uncertainty by characterizing outcomes with random variables. Random variables have distributions which are either discrete or continuous.
- In general, we observe samples; and use them to make inferences about a population (in practice, we tend to assume the population exists but it not available to us)
- We are concerned with the first four moments of a distribution:
    - Mean, typically denoted μ
    - Variance, the square of the standard deviation. Annualized standard deviation is called volatility; e.g., 12% volatility per annum. Variance is almost always denoted σ^2 and standard deviation by sigma, σ
    - Skew (a function of the third moment about the mean): a symmetrical distribution has zero skew or skewness
    - Kurtosis (a function of the fourth moment about the mean).
        - The normal distribution has kurtosis = 3.0
        - Excess kurtosis = 3 – Kurtosis. The *normal distribution,* being the benchmark, has excess kurtosis equal to zero
        - Kurtosis > 3.0 refers to a heavy-tailed distribution (a.k.a., **leptokurtosis**). Heavy-tailed distributions do tend to exhibit higher peaks, but our emphasis in risk is their heavy tails.
- The concepts of joint, conditional and marginal probability are important.
- To test a hypothesis about a sample mean (i.e., is the true population mean different than some value), we use a student t or normal distribution
    - Student t if the population variance is unknown (it usually is unknown)
    - If the sample is large, the student t remains applicable, but as it approximates the normal, for large samples the normal is used since the difference is not material
- To test a hypothesis about a sample variance, we use the chi-squared
- To test a joint hypothesis about regression coefficients, we use the F distribution
- In regard to the normal distribution:
    - N(mu, σ^2) indicates the only two parameters required. For example, N(3,10) connotes a normal distribution with mean of 3 and variance of 10 and, therefore, standard deviation of SQRT(10)
    - The **standard** normal distribution is N(0,1) and therefore requires no parameter specification: by definition it has mean of zero and variance of 1.0.
    - Please memorize, with respect to the standard normal distribution:
        - For N(0,1) →Pr(Z < -2.33) ~= 1.0% (CDF is one-tailed)
        - For N(0,1) → Pr (Z< -1.645)~ = 5.0% (CDF is one-tailed)

- The definition of a random sample is technical: the draws (or trials) are independent and identically distributed (i.i.d.)
    - Identical: same distribution
    - Independence: no correlation (in a time series, no autocorrelation)

- The assumption of i.i.d. is a precondition for:
    - Law of large numbers
    - Central limit theorem (CLT)
    - Square root rule (SRR) for scaling volatility; e.g., we typically scales a daily volatility of (V) to an annual volatility with V*SQRT(250). Please note that i.i.d. returns is the unrealistic precondition.

Probabilities (Miller Chapter 2)

P1.T2.708. Probability function fundamentals

Learning objectives: Calculate the probability of an event given a discrete probability function.

708.1. Let f(x) represent a probability function (which is called a probability mass function, p.m.f., for discrete random variables and a probability density function, p.d.f., for continuous variables) and let F(x) represent the corresponding cumulative distribution function (CDF); in the case of the continuous variable, F(X) is the integral (aka, anti-derivative) of the pdf. Each of the following is true about these probability functions **EXCEPT** which is false?

a) The limits of a cumulative distribution function (CDF) must be zero and one; i.e., F(-∞) = 0 and F(+∞) = 1.0
b) For both discrete and random variables, the cumulative distribution function (CDF) is necessarily an increasing function
c) In the case of a continuous random variable, we cannot talk about the probability of a specific value occurring; e.g., Pr[R = +3.00%] is meaningless
d) Bayes Theorem can only be applied to discrete random variables, such that continuous random variables must be transformed into their discrete equivalents

708.2. Consider a binomial distribution with a probability of each success, p = 0.050, and that total number of trials, n = 30 trials. What is the inverse cumulative distribution function associated with a probability of 25.0%?

a) Zero successes
b) One successes
c) Two successes
d) Three successes

708.3. For a certain operational process, the frequency of major loss events during a one period year varies form zero to 5.0 and is characterized by the following discrete probability mass function (pmf) which is the exhaustive probability distribution and where (b) is a constant:

Loss Events	pmf
X	f(X)
0	12b
1	7b
2	5b
3	3b
4	2b
5	1b

Which is nearest to the probability that next year **LESS THAN** two major loss events will happen?

a) 5.3%
b) 22.6%
c) 63.3%
d) 75.0%

Answers:

708.1. D. False. Bayes applies to both, although practicing applications are almost always using simple discrete random variables.

In regard to (A), (B) and (C), each is TRUE.

- ***In regard to true (B),*** the discrete CDF is an increasing step function.
- ***In regard to true (C),*** we need to specify an interval; e.g., Pr[2.95% < R < 3.10%]

708.2. B. One success. Binomial Pr(X = 0 successes) = 21.46% and Pr(X = 1 success) = 33.89% such that Pr(X ≤ 1) = 21.46% + 33.89% = 55.35%, and the cumulative 25.0% falls at one success; i.e., =BINOM.INV(30, 0.050, 0.250)

708.3. C. 63.3%. The sum of the pmf probabilities must be 100.0% such that 30*b = 1.0 or b = 1/30. Therefore the Pr [X < 2] = Pr[X ≤ 1] = Pr[X = 0] + Pr[X = 1] = 12/30 + 7/30 = 19/30 = 63.33%.

Discuss here in forum: *https://www.bionicturtle.com/forum/threads/p1-t2-708-probability-function-fundamentals-miller-ch-2.10766/*