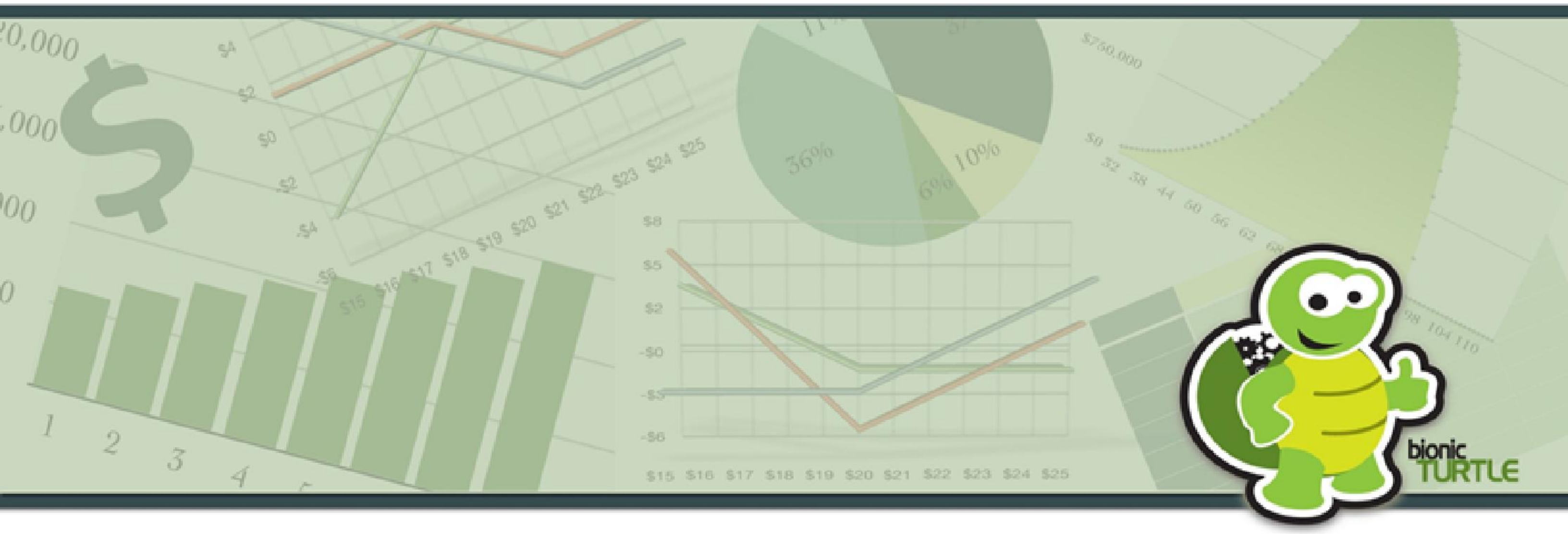

P1.T2. Quantitative Analysis

Bionic Turtle FRM Practice Questions

Chapter 4: Multivariate Random Variables

This is a super-collection of quantitative practice questions. It represents several years of cumulative history mapped to the current reading. Previous readings include Miller, Stock, and Gujarati, which we have retained in this practice question set.

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Note that this pertains to Chapters 1-6 in Topic 2, Quantitative Analysis. We will include this introduction in each of those practice question sets for reference.

Within each chapter, our practice questions are sequenced in *reverse chronological order* (appearing first are the questions written most recently). For example, consider Miller's Chapter 2 (Probabilities), you will notice there are fully three (3) sets of questions:

- Questions T2.708 to 709 (Miller Chapter 2) were written in 2017. The **7**XX denotes 2017.
- Questions T2.300 to 301 (Miller Chapter 2 were written in 2013. The **3**XX denotes 2103.
- Questions T2.201 & 204 (Stock & Watson) were written in 2012. Relevant but optional.

The reason we include the prior questions is simple: although the FRM's econometrics readings have churned in recent years (specifically, for Probabilities and Statistics, from Gujarati to Stock and Watson to Miller and now to GARP), the **learning objectives (AIMs) have remained essentially unchanged**. The testable concepts themselves, in this case, are generally quite durable over time.

Therefore, do not feel obligated to review all of the questions in this document! Rather, consider the additional questions as merely a *supplemental, optional* resource for those who want to spend additional time with the concepts.

The major sections are:

- **Fundamentals of Probabilities (current QA-1, Chapter 1)**
  - Most Recent BT questions, (20.1 and 20.2)
  - Previous BT questions, Miller Chapter 2 (T2.708 & T2.709)
  - Previous BT questions, Miller Chapter 2 (T2.300 & T2.301)
  - Previous BT questions, Stock & Watson Chapter 2 (T2.201 & T2.204)
  - Previous BT questions, Gujarati (T2.59 to T2.61, T2.65)

- **Random Variables (current QA-2, Chapter 2)**
  - Most Recent BT questions (20.3 and 20.4)
  - Previous BT questions, Miller Chapter 3 (T2.710 & T2.712)
  - Previous BT questions, Miller Chapters 2 & 3 (T2.303 & T2.307)
  - Previous BT questions, Gujarati (T2.58, T2.59, T2.62, T2.65 & T2.66)

- **Common Univariate Random Variables (current QA-3, Chapter 3)**
  - Most Recent BT questions, (20.5 to 20.7)
  - Previous BT questions, Miller Chapter 4 (T2.309 to T2.312 & T2.713 to T2.716)
  - Previous BT questions ,Stock & Watson Chapter 2 (T2.205)
  - Previous BT questions, Rachev Chapters 2 & 3 (T2.110 to T2.126)
  - Previous BT questions, Gujarati (T2.59, T2.68, T2.72 to T2.74, T2.82)

- **This Chapter: Multivariate Random Variables (current QA-4, Chapter 4)**
  - Most Recent BT questions Chapter 4 (20.8 to 20.10)
  - Previous BT questions, Miller Chapters 2, 3 & 4 (T2.304, T2.709, T2.711 & T2.716)
  - Previous BT questions, Stock & Watson Chapters 2 & 3 (T2.202, T2.212 to T2.213)
  - Previous BT questions, Gujarati (T2.62, T2.64, T2.65 & T2.67)

- **Sample Moments (current QA-5, Chapter 5)**
  - Most Recent BT questions, Miller Chapter 3 (T2.710 to T2.712)
  - Previous BT questions, Miller Chapter 3 (T2.303 to T2.308)
  - Previous BT questions, Stock & Watson Chapters 2 & 3 (T2.203,T2.206 to T2.208, T2.213)
  - Previous BT questions, Gujarati (T2.66, T2.67, T2.69 to T2.71)

- **Hypothesis Testing & Confidence Intervals (current QA-6, Chapter 6)**
  - Most Recent BT questions, Miller Chapter 7 (T2.718 & T2.719)
  - Previous BT questions, Miller Chapter 5 (T2.313 – T2.315)
  - Previous BT questions, Stock & Watson Chapter 3 (T2.209 to T2.212)
  - Previous BT questions, Gujarati (T2.75, T2.77, T2.79 to T2.81)

Probabilities - Key Ideas

- Risk measurement is largely the quantification of uncertainty. We quantify uncertainty by characterizing outcomes with random variables. Random variables have distributions that are either discrete or continuous.
- In general, we observe samples; and use them to make inferences about a population (in practice, we tend to assume the population exists but it not available to us)
- We are concerned with the first four moments of a distribution:
  - Mean, typically denoted μ
  - Variance, the square of the standard deviation. Annualized standard deviation is called volatility; e.g., 12% volatility per annum. Variance is almost always denoted σ^2 and standard deviation by sigma, σ
  - Skew (a function of the third moment about the mean): a symmetrical distribution has zero skew or skewness
  - Kurtosis (a function of the fourth moment about the mean).
    - The normal distribution has kurtosis = 3.0
    - Excess kurtosis = 3 – Kurtosis. The *normal distribution,* being the benchmark, has excess kurtosis equal to zero
    - Kurtosis > 3.0 refers to a heavy-tailed distribution (a.k.a., **leptokurtosis**). Heavy-tailed distributions do tend to exhibit higher peaks, but our emphasis in risk is their heavy tails.
- The concepts of joint, conditional, and marginal probability are important.
- To test a hypothesis about a sample mean (i.e., is the true population mean different than some value), we use a student t or normal distribution
  - Student t if the population variance is unknown (it usually is unknown)
  - If the sample is large, the student t remains applicable, but as it approximates the normal, for large samples the normal is used since the difference is not material
- To test a hypothesis about a sample variance, we use the chi-squared
- To test a joint hypothesis about regression coefficients, we use the F distribution
- In regard to the normal distribution:
  - N(mu, σ^2) indicates the only two parameters required. For example, N(3,10) connotes a normal distribution with a mean of 3 and variance of 10 and, therefore, a standard deviation of SQRT(10)
  - The **standard** normal distribution is N(0,1) and therefore requires no parameter specification: by definition, it has a mean of zero and variance of 1.0.
  - Please memorize, with respect to the standard normal distribution:
    - For N(0,1) →Pr(Z < -2.33) ~= 1.0% (CDF is one-tailed)
    - For N(0,1) → Pr (Z< -1.645)~ = 5.0% (CDF is one-tailed)

- The definition of a random sample is technical: the draws (or trials) are independent and identically distributed (i.i.d.)
    - Identical: same distribution
    - Independence: no correlation (in a time series, no autocorrelation)

- The assumption of i.i.d. is a precondition for:
    - Law of large numbers
    - Central limit theorem (CLT)
    - Square root rule (SRR) for scaling volatility; e.g., we typically scale a daily volatility of (V) to an annual volatility with V*SQRT(250). Please note that i.i.d. returns are the unrealistic precondition.

Chapter 4: Multivariate Random Variables

P1.T2.20.8. Probability matrix

Learning objectives: Explain how a probability matrix can be used to express a probability mass function. Compute the marginal and conditional distributions of a discrete bivariate random variable. Explain how the expectation of a function is computed for a bivariate discrete random variable.

20.8.1. A stock (X1) has three possible returns: -5%, 0, or +5%. The analyst rating (X2) can be negative (denoted by X2= -1), neutral (X2 = 0), or positive (X2 = +1). The probability matrix is displayed below (inside the square). Six joint probabilities are given, but three are missing; for example, the joint probability of a negative analyst rating and a negative stock return, P(X1 = -5% ∩ X2 = -1) = 14.0%.

		Stock Return (X1)		
		-5%	0%	5%
Analyst	Negative, -1	14.0%	10.0%	3.0%
Rating	Neutral, 0	4.0%	15.0%	15.0%
(X2)	Positive. +1			
	f(X1)	20.0%	34.0%	46.0%

The bottom row (outside the square) displays the unconditional (aka, marginal) probabilities for the stock; for example, the unconditional Pr(X1 = -5%) = 20.0%. What is the unconditional (aka, marginal) probability that the analyst rating is positive, Pr(X2 = +1)?

a) 28.0%
b) 39.0%
c) 46.0%
d) 60.9%

20.8.2. The probability matrix below gives the joint probabilities for two variables, X1 and X2. The stock (X1) has three possible returns: -5%, 0, or +5%. The analyst rating (X2) can be negative (denoted by X2= -1), neutral (X2 = 0) or positive (X2 = +1). Also given are the marginal (aka, unconditional) probabilities for the stock (X1), as follows: Pr(X1 = -5%) = 20%, Pr(X1 = 0%) = 50% and Pr(X1 = +5%) = 30%.

		Stock Return (X1)		
		-5%	0%	5%
Analyst	Negative, -1	10.0%	10.0%	2.0%
Rating	Neutral, 0	7.0%	15.0%	8.0%
(X2)	Positive. +1	3.0%	25.0%	20.0%
		20.0%	50.0%	30.0%

What is the probability that the stock return is non-negative conditional on a positive analyst rating; i.e., what is conditional Pr[(X1 = 0%) ∪ (X1 = +5%) | X2 = Positive]?

a) 45.00%
b) 66.67%
c) 75.00%
d) 93.75%

20.8.3. Below is a joint probability matrix for the variables X1 and X2.

		X1	
		1	2
X2	3	20.0%	30.0%
	4	35.0%	15.0%

What is the expected value of X1^X2?

a) 0.640
b) 3.671
c) 5.350
d) 6.500

Answers:

20.8.1. B. True: 39.0%. See below.

		Stock Return (X1)			
		-5%	0%	5%	f(X2)
Analyst Rating	Negative, -1	14.0%	10.0%	3.0%	27.0%
	Neutral, 0	4.0%	15.0%	15.0%	34.0%
	Positive. +1	2.0%	9.0%	28.0%	39.0%
		20.0%	34.0%	46.0%	100.0%

20.8.2. D. True: 93.75%

We only need to add the joint probabilities: P(X1 = 0% ∩ X2 = +1) + P(X1 = 5% ∩ X2 = +1) = 25.0% + 20.0% = 45.0%.

Then the conditional Pr[(X1 = 0%) ∪ (X1 = +5%) | X2 = Positive] = 45.0% / 48.0% = 93.750%

		Stock (S)			
		-5%	0%	5%	f(X2)
Analyst Rating	Negative, -1	10.0%	10.0%	2.0%	22.0%
	Neutral, 0	7.0%	15.0%	8.0%	30.0%
	Positive. +1	3.0%	25.0%	20.0%	48.0%
		20.0%	50.0%	30.0%	100.0%

20.8.3. C. True: 5.350

Because 1^3*20.0% + 2^3*30.0% + 1^4*35.0% + 2^4*15.0% = 0.20 + 2.40 + 0.35 + 2.40 = 5.350

Discuss here in the forum: *https://www.bionicturtle.com/forum/threads/p1-t2-20-8-probability-matrix.23340/*