P1.T3. Financial Markets and Products

Chapter 10. Pricing Financial Forwards and Futures

Bionic Turtle FRM Practice Questions

By David Harper, CFA FRM CIPM
www.bionicturtle.com

Key Ideas According to David

Chapter 10. Pricing Financial Forwards and Futures

Chapter 10. Pricing Financial Forwards and Futures

P1.T3.716. Arbitrage and the cost of carry model

Learning objectives: Differentiate between investment and consumption assets. Define short-selling and calculate the net profit of a short sale of a dividend-paying stock. Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices. Calculate the forward price given the underlying asset's spot price, and describe an arbitrage argument between spot and forward prices. Explain the relationship between forward and futures prices.

716.1. In April 2017, Portfolio Manager Jeff believed that the shares of Chorizo Fast Casual Restaurants Incorporated ("Chorizo Inc"), trading at $380.00 per share, were over-valued. He instructed his broker to short 1,000 shares. In May, Chorizo Inc paid a dividend of $6.00 per share. In July, due to a food contamination incident, the shares plummeted to $$333.00, when Jeff closed out his position. The total borrowing fee was $1,500.00. What was Jeff's net profit?

a) $39,500
b) $45,000
c) $47,000
d) $51,500

716.2. An investment asset has a current price of $60.00 while the risk-free rate is 3.0% per annum with continuous compounding. The asset pays income twice a year and the income is equal to 5.0% of the asset price at the time of income payment; in other words, the asset's yield is 10.0% per annum with semi-annual compounding. If a one-year forward contract on the asset has a price of $58.00, and if we make the typical theoretical cost of carry assumptions (e.g., no trading transaction costs), then which of the following best summarizes the arbitrage opportunity? [note: inspired by Hull's Example 5.3]

a) Cash and carry will realize $1.92 in future profit
b) Cash and carry will realize $2.44 in future profit
c) Reverse cash and carry will realize $3.83 in future profit
d) There is no arbitrage opportunity: the forward is not mis-priced

716.3. The spot price of wheat is $5.00 per bushel while the risk free-rate is 3.0% per annum with continuous compounding. The cost to store wheat is 12.00% per annum as a proportion of the spot price. The traded (observed) price of a nine-month wheat futures contract, F(0, 0.75), is $5.430. Among the following choices, which of the following scenarios is the most likely?

a) Arbitragers will immediately compel the futures contract price to increase by $0.165
b) Arbitragers will immediately compel the futures contract price to increase by $1.370
c) We can currently conduct a cash-and-carry arbitrage, borrowing to buy the asset at the current spot price, for a future profit of about $0.316 per bushel
d) The market is concerned that a possible shortage (i.e., lack of supply) in wheat might occur and this is reflected in a convenience yield of about 4.0% per annum as a proportion of the spot price

Answers:

716.1. A. $39,500.

In April, Jeff borrowed 1,000 shares and sold them for $380.00 each, which produced a cash inflow of +$380,000. As the investor who borrowed the shares, he PAID the May divided of $6.00 * 1,000 = $6,000 (cash outflow). When he closed out the position in July, he bought 1,000 shares for $333.00 each, so he paid -$333,000. When we include his borrowing cost, the net profit = +380,000 - $6,000 - 333,000 - 1,500 = $39,500. Equivalently, his per-share gain was (+380.0 - 6.0 - 333.0) = $41.0 per share, or $41,000, minus the borrowing fee of $1,500 equals $39,500.

716.2. A. Cash and carry will realize $1.92 in future profit

The investment's per annum yield with continuous compounding is equal to 2*LN(1+0.10/2) = 9.7580% such that the theoretical forward price should be $60.00*exp[(3.0% - 9.7580%)*1.0 year] = $56.0792. Because the observed futures price is "trading rich" at a higher $58.00, the arbitrage opportunity is a "cash and carry." If the futures price is trading too rich, that is if F(0) > theoretical price implied by cost of carry, then the arbitrage is cash and carry (as in, carry the commodity which is "trading cheap"): borrow to buy the asset and sell forward the asset with a short futures contract. If the futures price is trading too cheaply, that is if F(0) < theoretic price implied by COC, then the arbitrage is reverse cash and carry (short the commodity, invest the cash as the risk-free rate and buy the asset forward with a long futures position). In this case, the future profit will created by a "cash and carry" will be $58.00 - $56.0792 = $1.92.

Specifically:

- At time 0, we borrow $60.00 to buy the asset; i.e., zero initial cash outlay
- In one year, we deliver the asset per the short forward contract, receiving $58.00, and we repay the loan with $60.00*exp(3%*1) = $61.83. But we also received the income, which had a present value $60*[exp(-9.7580%*1)-1] = $5.58 and a future value of $5.58*exp(3%*1) = $5.75. So the final cash flow is a profit of +58.00 - $61.83 + 5.75 = $1.92, which is equal to the difference between the original forward price ($58.00) and the theoretical price implied by the cost of carry model ($56.0792).

716.3. D. The market is concerned that a possible shortage (i.e., lack of supply) in wheat might occur and this is reflected in a convenience yield of about 4.0% per annum as a proportion of the spot price.

According to Hull, a key difference between and investment commodity and a consumption commodity is the convenience yield engendered by ownership of the consumption commodity. According to Hull, "This argument [i.e., the two-sided no arbitrage argument] cannot be used for a commodity that is a consumption asset rather than an investment asset. Individuals and companies who own a consumption commodity usually plan to use it in some way. They are reluctant to sell the commodity in the spot market and buy forward or futures contracts, because forward and futures contracts cannot be used in a manufacturing process or consumed in some other way. There is therefore nothing to stop equation (5.14) from holding, and all we can assert for a consumption commodity is $F(0) \leq [S(0) + U]*\exp(rT)$." In the model, this is realized with the convenience yield such that, $F(0) = S(0)*\exp[(r+u-y)*T]$, where (y) is the convenience yield which "simply measures the extent to which the left-hand side is less than the right-hand side." Further, says Hull:

"For investment assets the convenience yield must be zero; otherwise, there are arbitrage opportunities ... The convenience yield reflects the market's expectations concerning the future availability of the commodity. The greater the possibility that shortages will occur, the higher the convenience yield. If users of the commodity have high inventories, there is very little chance of shortages in the near future and the convenience yield tends to be low. If inventories are low, shortages are more likely and the convenience yield is usually higher."

In regard to choice (A), this is actually a fine choice because $1.65 is the arbitrage profit if there is no convenience yield. Specifically, if y = 0, then $F(0) = \$5.00*\exp[(3.0\% + 12\%)*9/12] = \$5.595$, which is about $1.62 higher than the observed $5.430. (if this were the case, the arbitrage to exploit is a "reverse cash and carry" that shorts the expensive commodity and takes a long position in the futures contract).

Discuss here in forum: *https://www.bionicturtle.com/forum/threads/p1-t3-716-arbitrage-and-the-cost-of-carry-model-hull-chapter-5.10601/*