P1.T3. Financial Markets and Products

Chapter 15. Exotic Options

Bionic Turtle FRM Practice Questions

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Key Ideas According to David

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Hull Textbook Questions and Answers – Ch.26 (Selected)

Key Ideas According to David

Before 2020, these questions were covered by Hull, Options, Futures & Other Derivatives. Although GARP has separated the chapters into different readings, we are retaining these key ideas in each of the corresponding documents.

- Interest rates
- Futures/Forwards (Commodities)
- Interest rate futures
- Corporate Bonds
- Swaps
- Options and option trading strategies

Interest Rates

Three skills will put you in a good position: compound frequencies; present value (bond) pricing based on discounted cash flow; and implied forward rates given spot rates:

1. You **do need to be fluent with compound frequencies**. Probably, like the last exam, the default compound frequency will be annual. However, you still need to be ready to convert. If a rate is 8.0% per annum with annual compounding, you should easily be able to convert to its semi-annual and continuous equivalents.

2. Probably the most basic pricing skill is "vanilla" bond pricing; by vanilla, I refer to a basic coupon-bearing or zero-coupon bond. For example, given a zero rate curve (5% @ 0.5 years, 5.8% at 1.0 year, 6.4% at 1.5 years, and 6.8% at 2.0 years, each continuously compounding), what is the price of a two-year $100 face bond that pays a semi-annual coupon at a coupon rate of 6.0%. You should be able to do this.

3. ***GARP likes to test the implied forward rate given the spot rate curve***. You can almost expect to be asked. For example, if the 2-year spot rate is 1.2% and the 3-year spot rate is 1.4%, you should be able to infer the one-year forward rate, f(2,3), under continuous, annual and/or semi-annual compound frequencies.
    - Please note that GARP like a price-based variation on the implied forward rate, which I reviewed here at *http://www.bionicturtle.com/forum/threads/shortcut-to-forward-rates-if-you-have-bond-prices.4927/*

Futures/Forwards (Commodities)

This is a potentially deep topic. But I think your focus should be on the following:

- Cost of carry; i.e., be facile with computing the implied model forward price. But please do not only practice solving for F(0). You must be sufficiently comfortable such that you can, for example, extract the convenience yield if given F(0).
- The minimum variance hedge ratio is extremely likely to be tested; I included two examples in the FR, given it appears every year
- Please practice the optimal hedge (minimum variance) with both commodities and equity portfolios (hedged with index futures)
- I think margin accounts are testable (initial and maintenance margins for the futures positions that are used to hedge)
- With respect to futures contract sizes, I think you should know that T-bond futures are standardized at $100,000; Eurodollars at $1,000,000; and S&P 500 at a 250 multiple (*250). They are likely to be provided, but are common enough that it helps to just know them. More detail here at *http://www.bionicturtle.com/forum/threads/futures-contract-sizes.4959/*
- Be comfortable with contango/backwardation (observed) and normal contango/backwardation (unobserved)

Do you need to memorize the size of commodity contracts?
Probably an exam question will provide you with contract size, rather than assume you know. Although, I do think it is good practice to know the following due to their exam popularity:

- Treasury bonds: $100,000 (GARP may assume you know)
- S&P 500: $250 * index futures price (popularly used for questions)
- Eurodollar: $1,000,000

And the following are not uncommon:

- Gold: 100 troy ounces (I agree with you)
- NASDAQ 100: $100 * index futures price
- S&P & NASDAQ MINI contracts: one-fifth (1/5th); i.e., $50* and $20*
- Crude oil: 1,000 barrels
- Silver: 5,000 ounces (maybe, do most people know this? I don't think so....)
- Corn (= wheat): 5,000 bushels (popular in quizzes)
- Copper: 25,000 pounds

Where can you find these? *http://www.cmegroup.com/* e.g., *http://www.cmegroup.com/trading/metals/base/copper_contract_specifications.html*

Interest rate futures

I think you should focus on:

- Day count conventions
- Understanding the mechanics of the Eurodollar futures contract and Treasury bond futures contracts
- GARP likes to test cheapest-to-deliver (CTD); i.e., given three or four eligible bonds, identify the CTD
- Definitely be ready to compute the number of interest rate futures contracts used to duration hedge a fixed-income position. If you are given two durations, you do NOT want to hedge with the current durations, but RATHER the expected forward durations at maturity.

Swaps

I think the assignment (Hull) divides into three: comparative advantage; swap mechanics; and swap valuation. We have several practice questions on comparative advantage (it reduces to the observation that the total net gain equals the difference between fixed and floating rate differentials), but historically this tricky idea has barely been tested to my knowledge. You clearly need to be comfortable with swap mechanics so you can answer a very basic, non-quantitative question like one I included in the Focus Review (FR):

GARP 2010.P1.12. The yield curve is upward sloping, and a portfolio manager has a long position in 10-year Treasury Notes funded through overnight repurchase agreements. The risk manager is concerned with the risk that market rates may increase further and reduce the market value of the position. What hedge could be put on to reduce the position's exposure to rising rates?

a) Enter into a 10-year pay fixed and receive floating interest rate swap.
b) Enter into a 10-year receive fixed and pay floating interest rate swap.
c) Establish a long position in 10-year Treasury Note futures.
d) Buy a call option on 10-year Treasury Note futures.

Some key (exam) points to keep in mind with respect to swap mechanics:

- The vanilla interest rate swap (IRS) references notional; i.e., the notional is not exchanged (But the principal is exchanged in a currency swap, hence the maximum potential future [credit] exposure of a currency swap occurs at maturity)
- By default, the floating rate is determined at the beginning of each period and paid at the end; e.g., the first fixed-rate settlement is known at swap inception
- The duration of a swap position can be inferred from its valuation treatment as consisting of two bond legs: just as value[swap, POV of fixed-rate receiver, floating-rate payer] = value[fixed-rate bond] - value[floating-rate bond], the duration of the IRS from the perspective of the fixed-rate receiver (who is effectively long the fixed-rate bond-equivalent and short the floater) is approximately equal to the duration of the fixed-rate bond-equivalent. For example, the (modified) duration of a swap with a 3-year tenor, from the perspective of a 4.0% fixed rate payer is about 2.8 years at settlement because the duration equals 2.8 years (i.e., fixed rate bond) minus about zero (duration of floating-rate bond is time-to-next-coupon).

In regard to swap valuation, you must practice a few. You'll quickly see that it's just like pricing a bond but with a tiny additional step, where the key insight is that the floating-rate bond-equivalent, for valuation purposes, only requires a single cash flow due to the elegant fact that it prices exactly at par at the next settlement. In the FR, I included the classic sort of swap valuation that you could see on the exam:

GARP 2011.P1.E1.10. A bank had entered into a 3-year interest rate swap for a notional amount of USD 300 million, paying a fixed rate of 7.5% per year and receiving LIBOR annually. Just after the payment was made at the end of the first year, the continuously compounded 1-year and 2-year annualized LIBOR rates were 7% per year and 8% per year, respectively. The value of the swap at that time was closest to which of the following choices?

Options and option trading strategies

In collecting the three-year sample of exam-type questions, I was surprised at the high prevalence of put-call parity in the FRM. Historically, put-call parity questions are very common. (Please note this is a T3 summary and does not include discussion of option pricing models, OPM, which are T4 topics). It is essential that you memorize, and are utterly comfortable with, the put-call parity formula; for example, can you, without any reference, quickly produce the formula's equivalent of a covered call or protective put?

After you have mastered the usage of the put-call parity, c + K*exp(-rT) = p + S, you might take a look at my method for dealing with an arbitrage exploitation question, see *http://www.bionicturtle.com/forum/threads/how-to-work-put-call-parity-arbitrage-problems.6167/*

Finally, I would be familiar with Hull's rules about the optimality of early exercise under the four permutations of call/put and European/American.

Option Trading Strategies

In my opinion, the section (a single Hull chapter) requires some of your time, if you want to be fully ready. So far, it's always been included in the exam. And, as i mentioned in the FR audio, to illustrate how we lack a shortcut here, last year GARP asked a question about box spreads, which totally surprised me as it's a really minor strategy. With respect to mechanics, Hull parses them into:

- Asset + option; e.g., protective put, covered call
- Spread strategies
- Combinations

While that is a fine way to grasp them, you are unlikely to encounter an exam question along these lines. Rather, you want to focus on applications and risk/reward perspective, with particular emphasis on upside/downside potential. For example,

- Which of the strategies are long volatility?
- Which of the strategies are directional;
  i.e., benefit from an increase/decrease in asset price?
- Which have capped or uncapped payouts?
- Which produce an initial cash inflow?

Chapter 15. Exotic Options

P1.T3.729. Gap, forward start and compound exotic options

Learning objectives: Identify and describe the characteristics and pay-off structure of the following exotic options: Gap, Forward start & Compound.

729.1 Consider an asset with a current price of \$120.00 and volatility of 16.0% while the risk-free rate is 3.0%. A regular (aka, vanilla) but deeply out-of-the-money (OTM) one-year European put option on the stock with a strike price of \$100.00 has a price of \$0.740; i.e., p(S = \$120.00, K = \$100.00, σ = 0.160, Rf = 0.030, T = 1.0 year) = \$0.740.

Now consider the modification of this regular put option into a gap put option with the addition of a trigger price, denoted K2. In this case, the price of the gap put option is given by p(S = \$120.00, K1 = \$100.00, K2 = trigger price, σ = 0.160, Rf = 0.030, T = 1.0 year). Each of the following statements about this gap option is true EXCEPT which is false?

a) If K1 = K2 = \$100.00, then the gap option also has a price of \$0.740
b) If the strike price, K1= \$100.00 and the trigger price, K2 = \$110.00, then the gap option has a negative price
c) If the strike price, K1 = \$100.00 and the trigger price, K2 = \$90.00, then the gap option price is greater than \$0.740
d) Given a strike price, K1 = 100.00, among various trigger prices, the gap option has its highest price when the trigger, K2 = \$100.00

729.2. The exhibit below shows the call option prices for various times to maturity, T = {three months, six months, nine months, 1.0 year, 1.25 years, and 1.5 years} for an at-the-money European call option while the stock and strike price are both $100.00, the stock's volatility (σ) is 30.0%, the risk-free rate is 4.0% and the stock pays a continuous dividend yield of 7.0%:

European call option prices with various times to maturity (T)						
Stock (S0)	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
Strike (K)	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
Volatity, σ	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Riskfree rate, r	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
Time to maturity, T, years	0.25	0.50	0.75	1.00	1.25	1.50
Div Yield, q	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%
Implied PV lump-sum, D	$1.75	$3.51	$5.27	$7.04	$8.81	$10.58
BSM call price, c =	$5.53	$7.51	$8.88	$9.92	$10.76	$11.45

The stock price is $100.00 today. Consider a forward-start option that is a contract to buy, one year from today, a six-month to expiration at-the-money (ATM) call option; i.e., T1 = 1.0 year, T2 = 1.5 years. Which is nearest to the price of this forward-start option?

a) $7.00
b) $9.25
c) $10.68
d) $11.45

729.3. Consider a compound option that gives the holder the right to pay $1.90 in one year (T1) and purchase a call option with a strike price of $40.00 and one year to expiration; so this is a call-on-a-call with T1 = + 1.0 year and T2 = +2.0 years. The underlying stock price is currently $37.50. The stock's volatility is 30.0% and the riskfree rate is 4.0%. For convenience, the option prices of a one-year European call are shown below at stock prices according to $2.50 intervals:

European call option prices with stock prices						
Stock (S0)	$30.00	$32.50	$35.00	$37.50	$40.00	$42.50
Strike (K)	$40.00	$40.00	$40.00	$40.00	$40.00	$40.00
Volatity, σ	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Riskfree rate, r	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
Time to maturity, T, years	1.00	1.00	1.00	1.00	1.00	1.00
BSM call price, c =	$1.16	$1.90	$2.87	$4.08	$5.50	$7.13

Under what condition(s) will this compound call-on-a-call option be exercised in one year?

a) This compound call will never be exercised
b) This compound call will be exercised if the stock price, S(1.0), is above $32.50
c) This compound call will be exercised if the underlying call has positive intrinsic value at time T1; i.e., if S(1.0) >= $40.00
d) The compound call will be exercised if the underlying call has a lower bound of $1.90 at time T1; i.e., if S(1.0) >= $40.33

Answers:

729.1. C. False. Instead, the true statement is "If the strike price, K1 = $100.00 and the trigger price, K2 = $90.00, then the gap option price is less than $0.740" because in this case the actual gap option price is equal to $0.419.

- In regard to (A), (B) and (C), each is TRUE.
- In regard to true (B), if the trigger K2 = $110.00, then the gap put price = -$0.050.

729.2. A. TRUE: $7.00. The forward-start price is given by, c = $7.51*exp(-7%*1) = $7.0022

729.3. B. TRUE: This compound call will be exercised if the stock price, S(1.0), is above $32.50. If S(1.0) = $32.50, then the call option price will be $1.90, such that a higher future stock price implies the call option value is greater than the purchase price of $1.90 and the compound option will be worth exercising. Simple! Please note the initial value of this compound option is neither mentioned nor relevant.

Discuss here in forum: *https://www.bionicturtle.com/forum/threads/p1-t3-729-gap-forward-start-and-compound-exotic-options-hull-chapter-26.10733/*