



P1.T3. Financial Markets and Products

Chapter 16. Properties of Interest Rates

Bionic Turtle FRM Practice Questions

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Key Ideas According to David

Before 2020, these questions were covered by Hull, Options, Futures & Other Derivatives. Although GARP has separated the chapters into different readings, we are retaining these key ideas in each of the corresponding documents.

- Interest rates
- Futures/Forwards (Commodities)
- Interest rate futures
- Corporate Bonds
- Swaps
- Options and option trading strategies

Interest Rates

Three skills will put you in a good position: compound frequencies; present value (bond) pricing based on discounted cash flow; and implied forward rates given spot rates:

1. You **do need to be fluent with compound frequencies**. Probably, like the last exam, the default compound frequency will be annual. However, you still need to be ready to convert. If a rate is 8.0% per annum with annual compounding, you should easily be able to convert to its semi-annual and continuous equivalents.
2. Probably the most basic pricing skill is “vanilla” bond pricing; by vanilla, I refer to a basic coupon-bearing or zero-coupon bond. For example, given a zero rate curve (5% @ 0.5 years, 5.8% at 1.0 year, 6.4% at 1.5 years, and 6.8% at 2.0 years, each continuously compounding), what is the price of a two-year \$100 face bond that pays a semi-annual coupon at a coupon rate of 6.0%. You should be able to do this.
3. **GARP likes to test the implied forward rate given the spot rate curve**. You can almost expect to be asked. For example, if the 2-year spot rate is 1.2% and the 3-year spot rate is 1.4%, you should be able to infer the one-year forward rate, $f(2,3)$, under continuous, annual and/or semi-annual compound frequencies.

Please note that GARP like a price-based variation on the implied forward rate, which I reviewed here at <http://www.bionicturtle.com/forum/threads/shortcut-to-forward-rates-if-you-have-bond-prices.4927/>

Futures/Forwards (Commodities)

This is a potentially deep topic. But I think your focus should be on the following:

- Cost of carry; i.e., be facile with computing the implied model forward price. But please do not only practice solving for $F(0)$. You must be sufficiently comfortable such that you can, for example, extract the convenience yield if given $F(0)$.
- The minimum variance hedge ratio is extremely likely to be tested; I included two examples in the FR, given it appears every year
- Please practice the optimal hedge (minimum variance) with both commodities and equity portfolios (hedged with index futures)
- I think margin accounts are testable (initial and maintenance margins for the futures positions that are used to hedge)
- With respect to futures contract sizes, I think you should know that T-bond futures are standardized at \$100,000; Eurodollars at \$1,000,000; and S&P 500 at a 250 multiple (*250). They are likely to be provided, but are common enough that it helps to just know them. More detail here at <http://www.bionicturtle.com/forum/threads/futures-contract-sizes.4959/>
- Be comfortable with contango/backwardation (observed) and normal contango/backwardation (unobserved)

Do you need to memorize the size of commodity contracts?

Probably an exam question will provide you with contract size, rather than assume you know. Although, I do think it is good practice to know the following due to their exam popularity:

- Treasury bonds: \$100,000 (GARP may assume you know)
- S&P 500: \$250 * index futures price (popularly used for questions)
- Eurodollar: \$1,000,000

And the following are not uncommon:

- Gold: 100 troy ounces (I agree with you)
- NASDAQ 100: \$100 * index futures price
- S&P & NASDAQ MINI contracts: one-fifth (1/5th); i.e., \$50* and \$20*
- Crude oil: 1,000 barrels
- Silver: 5,000 ounces (maybe, do most people know this? I don't think so....)
- Corn (= wheat): 5,000 bushels (popular in quizzes)
- Copper: 25,000 pounds

Where can you find these? <http://www.cmegroup.com/> e.g., http://www.cmegroup.com/trading/metals/base/copper_contract_specifications.html

Interest rate futures

I think you should focus on:

- Day count conventions
- Understanding the mechanics of the Eurodollar futures contract and Treasury bond futures contracts
- GARP likes to test cheapest-to-deliver (CTD); i.e., given three or four eligible bonds, identify the CTD
- Definitely be ready to compute the number of interest rate futures contracts used to duration hedge a fixed-income position. If you are given two durations, you do NOT want to hedge with the current durations, but RATHER the expected forward durations at maturity.

Swaps

I think the assignment (Hull) divides into three: comparative advantage; swap mechanics; and swap valuation. We have several practice questions on comparative advantage (it reduces to the observation that the total net gain equals the difference between fixed and floating rate differentials), but historically this tricky idea has barely been tested to my knowledge. You clearly need to be comfortable with swap mechanics so you can answer a very basic, non-quantitative question like one I included in the Focus Review (FR):

GARP 2010.P1.12. The yield curve is upward sloping, and a portfolio manager has a long position in 10-year Treasury Notes funded through overnight repurchase agreements. The risk manager is concerned with the risk that market rates may increase further and reduce the market value of the position. What hedge could be put on to reduce the position's exposure to rising rates?

- a) Enter into a 10-year pay fixed and receive floating interest rate swap.
- b) Enter into a 10-year receive fixed and pay floating interest rate swap.
- c) Establish a long position in 10-year Treasury Note futures.
- d) Buy a call option on 10-year Treasury Note futures.

Some key (exam) points to keep in mind with respect to swap mechanics:

- The vanilla interest rate swap (IRS) references notional; i.e., the notional is not exchanged (But the principal is exchanged in a currency swap, hence the maximum potential future [credit] exposure of a currency swap occurs at maturity)
- By default, the floating rate is determined at the beginning of each period and paid at the end; e.g., the first fixed-rate settlement is known at swap inception
- The duration of a swap position can be inferred from its valuation treatment as consisting of two bond legs: just as $\text{value}[\text{swap, POV of fixed-rate receiver, floating-rate payer}] = \text{value}[\text{fixed-rate bond}] - \text{value}[\text{floating-rate bond}]$, the duration of the IRS from the perspective of the fixed-rate receiver (who is effectively long the fixed-rate bond-equivalent and short the floater) is approximately equal to the duration of the fixed-rate bond-equivalent. For example, the (modified) duration of a swap with a 3-year tenor, from the perspective of a 4.0% fixed rate payer is about 2.8 years at settlement because the duration equals 2.8 years (i.e., fixed rate bond) minus about zero (duration of floating-rate bond is time-to-next-coupon).

In regard to swap valuation, you must practice a few. You'll quickly see that it's just like pricing a bond but with a tiny additional step, where the key insight is that the floating-rate bond-equivalent, for valuation purposes, only requires a single cash flow due to the elegant fact that it prices exactly at par at the next settlement. In the FR, I included the classic sort of swap valuation that you could see on the exam:

GARP 2011.P1.E1.10. A bank had entered into a 3-year interest rate swap for a notional amount of USD 300 million, paying a fixed rate of 7.5% per year and receiving LIBOR annually. Just after the payment was made at the end of the first year, the continuously compounded 1-year and 2-year annualized LIBOR rates were 7% per year and 8% per year, respectively. The value of the swap at that time was closest to which of the following choices?

Options and option trading strategies

In collecting the three-year sample of exam-type questions, I was surprised at the high prevalence of put-call parity in the FRM. Historically, put-call parity questions are very common. (Please note this is a T3 summary and does not include discussion of option pricing models, OPM, which are T4 topics). It is essential that you memorize, and are utterly comfortable with, the put-call parity formula; for example, can you, without any reference, quickly produce the formula's equivalent of a covered call or protective put?

After you have mastered the usage of the put-call parity, $c + K \cdot \exp(-rT) = p + S$, you might take a look at my method for dealing with an arbitrage exploitation question, see <http://www.bionicturtle.com/forum/threads/how-to-work-put-call-parity-arbitrage-problems.6167/>

Finally, I would be familiar with Hull's rules about the optimality of early exercise under the four permutations of call/put and European/American.

Option Trading Strategies

In my opinion, the section (a single Hull chapter) requires some of your time, if you want to be fully ready. So far, it's always been included in the exam. And, as i mentioned in the FR audio, to illustrate how we lack a shortcut here, last year GARP asked a question about box spreads, which totally surprised me as it's a really minor strategy. With respect to mechanics, Hull parses them into:

- Asset + option; e.g., protective put, covered call
- Spread strategies
- Combinations

While that is a fine way to grasp them, you are unlikely to encounter an exam question along these lines. Rather, you want to focus on applications and risk/reward perspective, with particular emphasis on upside/downside potential. For example,

- Which of the strategies are long volatility?
- Which of the strategies are directional; i.e., benefit from an increase/decrease in asset price?
- Which have capped or uncapped payouts?
- Which produce an initial cash inflow?

Chapter 16. Properties of Interest Rates

P1.T3.712. Interest rate fundamentals
P1.T3.713. Spot and forward rates in bond pricing
P1.T3.714. Duration, modified duration and dollar duration
P1.T3.715. Par yield, convexity and term structure theories
P1.T3.157. Rate compounding frequency
P1.T3.158. Bond price using spot rates
P1.T3.159. Forward rates from spot (zero) rates
P1.T3.160. Valuation of forward rate agreement (FRA)
P1.T3.161. Bond duration & convexity
P1.T3.162. Interest rate term structure theories
P1.T3.163. Hull's interest rate definitions

P1.T3.712. Interest rate fundamentals

Learning Objectives: Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the “risk-free” rate. Calculate the value of an investment using different compounding frequencies. Convert interest rates based on different compounding frequencies.

712.1. Interest rates are a fundamental and essential building block in finance. As Hull reminds us, "for any given currency, many different types of interest rates are regularly quoted. These include mortgage rates, deposit rates, prime borrowing rates, and so on" and, further, interest rates are a factor in the valuation of virtually all derivatives. Consequently, fluency in finance requires that we achieve proficiency with the different types of rates and an ability to manipulate them depending on the circumstance. Each of the following statements is true **EXCEPT** which is false?

- a) A repo rate is essentially similar to the federal funds rate except that a repo rate can only be an overnight (i.e., one day) rate and will be slightly higher due to its additional credit risk
- b) Treasury rates are (i) the rates earned on instruments issued by a government to borrow in its own currency, (ii) are generally regarded as risk-free, but (iii) tend NOT to be the risk-free rate used to value derivatives
- c) The OIS rate is the agreed fixed rate in an overnight indexed swap (OIS). OIS rates are continually refreshed one-day rates, and subsequent to the Global Financial Crisis (GFC), OIS rates have been used as risk-free rates
- d) LIBOR (London Interbank Offered Rate) is an unsecured short-term borrowing rate between banks. LIBOR rates are quoted for a number of different currencies and borrowing periods that range from one day to one year. LIBOR rates are used as reference for hundreds of trillions of dollars of global transactions. However, LIBOR is suboptimal reference rate for derivatives transactions because it is determined from estimates made by banks, not from market transactions

712.2. Richard plans to invest \$10,000.00 today in a zero-coupon bond with a promised return of 7.0% per annum. This return is possible because he will not be repaid until the bond matures in ten (10) years. He calculates the future principal repayment, but his calculation assumes the rate is an equivalent annual interest rate; aka, effective annual rate. His advisor informs him that the actual rate is 7.0% per annum with monthly compounding. Compared to his original future value, how many dollars greater is Richard's revised future principal repayment?

- a) Zero
- b) \$39.80
- c) \$215.75
- d) \$425.10

712.3. If the six-year discount factor, $df(6.0)$, is \$0.820 then which of the following is nearest to the basis point difference between the implied semi-annual six-year spot rate and the implied continuously compounded six-year spot rate?

- a) Zero
- b) 2.75 basis points
- c) 30.0 basis points
- d) 249 basis points

Answers:

712.1. A. False. In contrast to the overnight federal funds rate (and LIBOR), repo rates are SECURED borrowing rates and will consequently will be (in general) slightly below the corresponding federal funds (or LIBOR) rate; further, although an overnight repo is most common, longer terms repos are used; aka, term repos.

In regard to (B), (C) and (D), each is TRUE.

712.2. D. \$425.10

If the rate is 7.0% per annum with annual compounding, the future value (principal repayment, in this case) is $\$10,000 * (1+7.0\%)^{10} = \$19,671.51$.

If the rate is 7.0% per annum with monthly compounding, the future value (principal repayment, in this case) is $\$10,000 * (1+7.0\%/12)^{(10*12)} = \$20,096.61$.

The difference is \$425.1002.

712.3. B. 2.75 basis points

Under semi-annual compound frequency, the implied six-year spot rate equals $2 * [(1/0.820)^{(1/12)} - 1] = 3.33502\%$. Under continuous compounding, the implied six-year spot rate equals $-1/6 * \ln(0.820) = 3.30752\%$. The difference is 0.02750% or about 2.75005 basis points.

Discuss here in forum: <https://www.bionicturtle.com/forum/threads/p1-t3-712-interest-rate-fundamentals-hull-chapter-4.10570/>