P1.T4. Valuation & Risk Models

Bionic Turtle FRM Practice Questions

Chapter 9. Pricing Conventions, Discounting, and Arbitrage

By David Harper, CFA FRM CIPM
www.bionicturtle.com

Chapter 9. Pricing Conventions, Discounting, and Arbitrage

P1.T4.900. Discount function and the Law of One Price 3
P1.T4.901. Exploiting arbitrage opportunities with a replicating bond portfolio (also: clean versus dirty, and day-count conventions)..... 7
P1.T4.309. Discount factors and law of one price 11
P1.T4.310. U.S. Treasury STRIPS 14
P1.T4.311. Accrued interest.....18
P1.T4.10. Time value of money.....20
P1.T4.11. Treasury STRIPS.....24

Chapter 9. Pricing Conventions, Discounting, and Arbitrage

P1.T4.900. Discount function and the Law of One Price
P1.T4.901. Exploiting arbitrage opportunities with a replicating bond portfolio (also: clean versus dirty, and day-count conventions)
P1.T4.309. Discount factors and law of one price
P1.T4.310. U.S. Treasury STRIPS
P1.T4.311. Accrued interest
P1.T4.10. Time value of money
P1.T4.11. Treasury STRIPS

P1.T4.900. Discount function and the Law of One Price

Learning objectives: Define discount factor and use a discount function to compute present and future values. Define the "law of one price," explain it using an arbitrage argument, and describe how it can be applied to bond pricing. Identify the components of a US Treasury coupon bond, and compare and contrast the structure to Treasury STRIPS, including the difference between P-STRIPS and C-STRIPS.

900.1. Displayed below (in the rightmost column) is a discount function implied by six U.S. Treasury bonds with various maturities. The bonds (one per row) pay a semi-annual coupon and mature at six-month intervals over the next three years:

Selected U.S. Treasury Bond Prices as of May 28, 2018

Time	Maturity	Coupon	Price	Discount Function (1)
0.50	11/30/2018	1.250%	$100.000	0.99380
1.00	5/31/2019	4.875%	$103.060	0.98240
1.50	11/30/2019	4.500%	$102.800	0.96190
2.00	5/31/2020	4.750%	$103.250	0.94040
2.50	11/30/2020	3.375%	$99.300	0.91220
3.00	5/31/2021	3.500%	$98.150	0.88220

(1) Discount factors rounded to four decimal places

Consider a new U.S. Treasury bond issued on May 28, 2018 with a maturity of three years that pays an annual coupon of 6.0% per annum. Unlike the Treasury bonds in the exhibit, this new bond pays coupons once per year and therefore has only three cash flows before maturing on 5/31/2021. Which is **nearest** to the bond's present value; aka, theoretical price?

a) $88.22
b) $97.34
c) $100.99
d) $105.05

900.2. Consider the following two U.S. Treasury bonds:

- Bond #1 has a remaining maturity of exactly five years, has a coupon rate of 2.0% per annum and pays a semi-annual coupon. Its current price is $81.600.
- Bond #2 has a remaining maturity of exactly five years, has a coupon rate of 7.0% per annum and pays a semi-annual coupon. Its current price is $104.050.

If we can assume the validity of the Law of One Price, then which of the following must be the price of a third bond (aka, Bond #3) that has a remaining maturity of exactly five years and a semi-annual (i.e., payable) coupon rate of 5.0% per annum?

a) $90.580
b) $95.070
c) $101.356
d) Need more information (specifically, the discount function)

900.3. In regard to Tuckman[1]'s discussion of the components and structure of U.S. Treasury STRIPS, which of the following statements is **TRUE**?

a) The holder of a U.S. Treasury STRIP is immunized against inflation risk but exposed to significant liquidity risk and call risk
b) The Law of One Price says an arbitrage profit is necessarily available when two securities offer identical cash flows but sell at different market prices
c) If we apply the discount function implied only by C-STRIPS (i.e., without P-STRIPS) to infer the price of a U.S. Treasury bond, we are likely to undervalue the security relative to its actual market price
d) Because C-STRIPS and P-STRIPS are perfect commodities, arbitrage enforces the law of one price to ensure that, with respect to U.S. Treasury bonds, theoretical (aka, model) prices equal market prices

[1] Bruce Tuckman, Fixed Income Securities, 3rd Edition (Hoboken, NJ: John Wiley & Sons, 2011)

Answers:

900.1. D. $105.05 because we can simply apply the discount factors: $6.00 * 0.98240 + $6.00 * 0.94040 + $106.00 * 0.88220 = $5.8944 + $5.6424 + $93.5132 = $105.050

900.2. B. $95.070 because (40% * $81.60) + (60% * 104.050) = $95.070; the third bond's cash flows are replicated (exactly matched) by a two-asset portfolio that includes 40% of the face value of Bond #1 plus 60% of the face value of Bond #2, therefore its price must be a 40/60% weighted average of those bond's prices. This is a common exam-type question and the key is to realize that the third bond's $5.00 annual coupon cash flow (i.e., $2.50 every six months) is matched if we combine 40% of the $2.00 coupon and 60% of the $7.00 coupon.

900.3. C. True: If we apply the discount function implied only by C-STRIPS (i.e., without P-STRIPS) to infer the price of a U.S. Treasury bond, we are likely to undervalue the security relative to its actual market price.

In regard to true (C), a key point raised by Tuckman is that individual Treasury bonds have idiosyncratic features that are reflected in their market prices. In particular, P-STRIPS are not fungible and therefore their prices inherit the idiosyncratic features of their associated bond issues (emphasis ours): "If U.S. Treasury bonds were commodities, with each regarded solely as a particular collection of cash flows, then the price of each would be well approximated by discounting its cash flows with the C-STRIPS discount factors [of Figure 1.1]. If however individual bonds have unique characteristics that are reflected in pricing, the law of one price would not be as accurate an approximation. **Furthermore, since C-STRIPS are fungible while P-STRIPS are not, any such pricing idiosyncrasies would manifest themselves as differences between the prices of P-STRIPS and C-STRIPS of the same maturity.**

... Inspection [of Figure 1.2] shows that there are indeed significant pricing differences between P-STRIPS and C-STRIPS that mature on the same date. This does not necessarily imply the existence of arbitrage opportunities, as discussed at the end of the previous section. However, the results do suggest that bonds have idiosyncratic pricing differences and that these differences are inherited by their respective P-STRIPS." About the idea that P-STRIPS are not fungible, Tuckman explains: "C-STRIPS are fungible while P-STRIPS are not. When reconstituting a bond, any C-STRIPS maturing on a particular date may be applied toward the coupon payment of that bond on that date. By contrast, only P-STRIPS that were stripped from a particular bond may be used to reconstitute the principal payment of that bond. This feature of the STRIPS program implies that P-STRIPS, and not C-STRIPS, inherit the cheapness or richness of the bonds from which they came ..."[2]

- continued on next page -

[2] Bruce Tuckman, Fixed Income Securities, 3rd Edition (Hoboken, NJ: John Wiley & Sons, 2011)

In regard to (A), (B) and (D), each is FALSE:

- ***In regard to false (A),*** STRIPS (Separate Trading of Registered Interest and Principal of Securities) are different than TIPS (Treasury Inflation Protected Securities). TIPS protect against inflation risk, but STRIPS are exposed to inflation risk. Further, STRIPS have no call risk and virtually no liquidity risk.
- ***In regard to false (B),*** a key idea explained by Tuckman is that violation of the Law of One Price creates an arbitrage opportunity but not all arbitrage opportunities can be exploited for profit. If the securities are indeed commodities, there exist at least two complications: "First, there are transaction costs in doing arbitrage trades which could significantly lower or wipe out any arbitrage profit... Second, bid-ask spreads in the financing markets (see Chapter 12), incurred when shorting securities, might also overwhelm any arbitrage profit.[3]"
- ***In regard to false (D),*** as explained in the quote above, U.S. Treasury bonds are commodities (i.e., fungible cash flow collections) only in theory. Maybe the most obvious example of this lack of fungibility is the observation that on-the-run U.S. Treasury bonds trade at a premium due to the superior liquidity features.

Discuss here in the forum: https://www.bionicturtle.com/forum/threads/p1-t4-900-discount-function-and-the-law-of-one-price-tuckman-ch-1.22065/

[3] Bruce Tuckman, Fixed Income Securities, 3rd Edition (Hoboken, NJ: John Wiley & Sons, 2011)