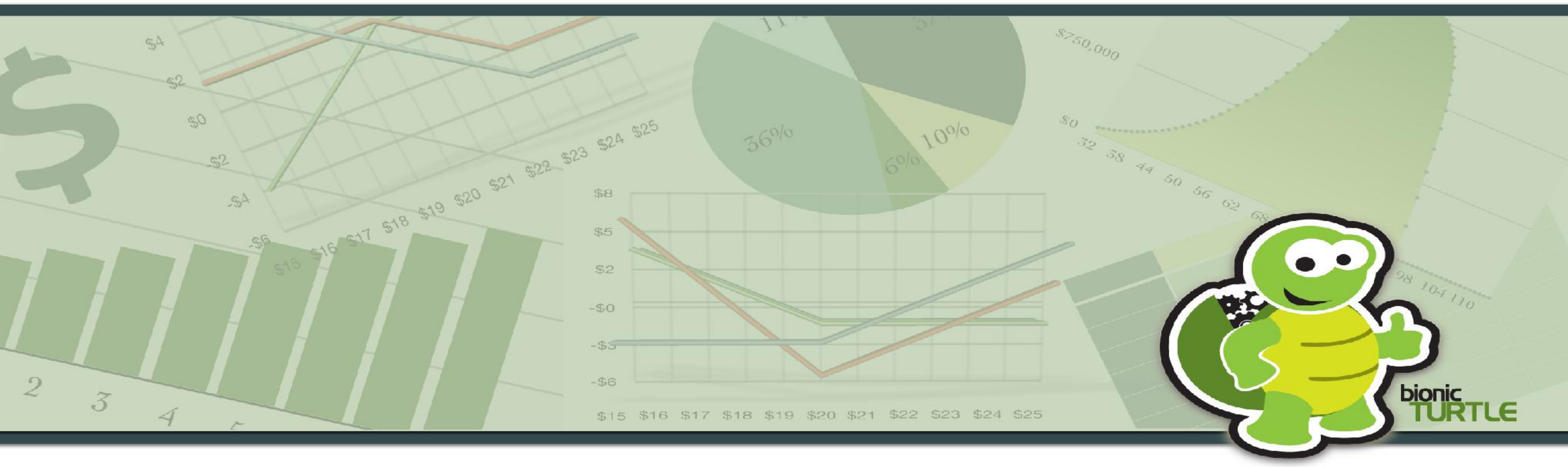

P1.T3. Financial Markets & Products

Chapter 15. Exotic Options

Bionic Turtle FRM Study Notes

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- **Define and contrast exotic derivatives and plain vanilla derivatives.**
- **Describe some of the factors that drive the development of exotic derivative products.**
- **Explain how any derivative can be converted into a zero-cost product.**
- **Describe how standard American options can be transformed into nonstandard American options.**
- **Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, Asian, exchange, and basket options.**
- **Describe and contrast volatility and variance swaps.**
- **Explain the basic premise of static option replication and how it can be applied to hedging exotic options.**

Define and contrast exotic derivatives and plain vanilla derivatives.

Plain vanilla derivatives are the standard European and American (call or put) options traded mostly on the exchanges. According to John Hull,[1]

- They are standardized options with contract specifications set by the exchanges.
- They are very active and popular exchange-traded products.
- Their prices or implied volatilities are set by exchanges or by interdealer brokers periodically.

Exotic derivatives (exotics) are non-standard options that are mostly traded over-the-counter.

- They are highly customized by financial engineers to tailor to the needs of the OTC investors.
- Derivates markets comprise only a smaller share of the exotics, yet they give potentially higher returns compared to plain vanilla products (due to wide bid-offer spreads).

Describe some of the factors that drive the development of exotic derivative products.

Some factors driving the development of exotics are:

- Exotics offer better hedging opportunities than plain vanilla products;
- They serve the investors for tax, accounting, legal, or regulatory purposes;
- Exotic products may reveal views regarding the likely market movement of several variables; they are well-suited to *speculative* bets.
- They may be structured to look more appealing to investors than they actually are.

[1] Hull, C. John, "Options, Futures and Other Derivatives", Ch: 26, "Exotic options", 10th Edition, Pearson, 2018.

Explain how any derivative can be converted into a zero-cost product.

A derivative can be converted into a zero-cost product by delaying the payment required for its initial cost until the expiration of the contract.

Usually, **packages** (a portfolio of plain vanilla options on an underlying - e.g., bull/bear spreads, butterfly spreads, calendar spreads, straddles, strangles, etc.) are designed in such a way that they have no initial cost.

- For example, a zero-cost collar, which comprises of a long call and a short put (or a short call and long put), can be constructed. This can be done by selecting the strike prices in such a way (i.e., strike price of call is higher than strike price of put) that the premium paid for the call is compensated by the premium collected from the put.

Any derivative can be transformed into a product with zero cost. For example, consider a European call option:[2]

- Cost of option when payment is initially made (Time= 0): $= \mathrm{c}$
- Cost of option when payment is made at maturity (Time =T): $= \mathrm{ce}^{\mathrm{rT}}$. Let this cost be defined as A.
- Thus the payoff when the option cost (A) is paid at maturity instead of at the time of entering into a contract, if $\mathrm{S_T}$ is the final asset price, and K is the strike price is:

$$\max(S_T - K, 0) - A \Rightarrow \max(S_T - K - A, -A)$$

So, this zero-cost derivative is similar to a forward contract as the initial cost of the product is zero. They can also be constructed as futures contracts in exchanges and are called futures style options, which are marked to market like any other futures contract.

In case the strike price, K, equals the forward price, then these zero costs products are known by other names such as break forward, Boston option, forward with optional exit, and cancelable forward.

Describe how standard American options can be transformed into nonstandard American options.

In contrast with standard American options traded in exchanges with a specified strike price and exercised any time prior to maturity, nonstandard American options are mostly traded in OTC markets and are transformed from the standard ones by:

- *Limiting early exercise to specific dates*: Such options can be exercised only on specific dates and are called Bermudan options. For example, some bond options are allowed to be exercised only on certain interest payment dates.
- *Allowing early exercise only for a part of the life of the option*: For example, employee stock options have an initial lockout period when they cannot be exercised.
- *Altering the strike price of the option*: For example, corporate bonds with embedded call options permit the issuer to retire the debt earlier, and the strike price for these bonds decreases over time.

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[2] Formulas retrieved from Hull, C. John, “Options, Futures and Other Derivatives”, Ch: 26, “Exotic options”, 10th Edition, Pearson, 2018.

- A warrant issued by a corporation on their own stock may constitute the above properties. For example[3], a warrant with seven years to maturity may be exercised only on specific dates from year 3 to 7 (initial lockout period and exercise limited to particular dates), with the strike price at \$30 in years 3 and 4, \$32 in year 5 and 6, and \$33 in the 7th year (different strike prices).

Just like the standard American options, non-standard American options can also be valued utilizing binomial trees.

Identify and describe the characteristics and pay-off structure of: Gap Options

A gap option is a European call or put option which has a strike price of K_1, which is used for calculating the payoff, and a trigger price of K_2, which is used for deciding whether the option makes the payoff or not. When the strike price equals the trigger price, the gap option is the same as an ordinary option.

Pay-off structure of gap options are:[4]

- A **gap call option** has a nonzero payoff (positive or negative) of $S_T - K_1$ when the final stock price exceeds the trigger price ($S_T > K_2$)
- A **gap put option** has a nonzero payoff of $K_1 - S_T$ if the final stock price is less than the trigger price ($S_T < K_2$)
- Gap options are valued using the Black-Scholes-Merton formula (non-dividend stock). The price for gap options is greater than the price of a regular call option (with strike price K_2) by: $(K_2 - K_1)e^{-rt}N(d_2)$

	Gap option	Regular option
Call	$S_0N(d_1) - K_1e^{-rt}N(d_2)$	$S_0N(d_1) - Ke^{-rt}N(d_2)$
Put	$K_1e^{-rt}N(-d_2) - S_0N(-d_1)$	$Ke^{-rt}N(-d_2) - S_0N(-d_1)$

Gap options: $d_1 = \frac{\ln(S_0/K_2)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$; $d_2 = d_1 - \sigma\sqrt{T}$

Hull Example 26.1:[5] The current price of a stock that pays no dividend is \$500,000 (S_0). Its volatility is assumed to be 20% for the next year. The risk-free rate is 5%. An insurance company has issued a **regular put option** where the policyholder has the right to sell the asset for \$400,000 (strike price) if the asset value falls below that level.

Let the cost of transferring the asset be \$50,000. If the policyholder is supposed to bear this cost, then this is a **gap put option**. It will be exercised only when the stock value is less than \$350,000. Here the strike price, K_1 is still \$400,000, but the trigger price, K_2, is \$350,000.

[3] Hull, C. John, "Options, Futures and Other Derivatives", Ch: 26, "Exotic options", 10th Edition, Pearson, 2018.
[4] Formulas retrieved from Hull, C. John, "Options, Futures and Other Derivatives", Ch: 26, "Exotic options", 10th Edition, Pearson, 2018.
[5] Hull, C. John, "Options, Futures and Other Derivatives", Ch: 26, "Exotic options", 10th Edition, Pearson, 2018. Spreadsheet handcrafted by David.