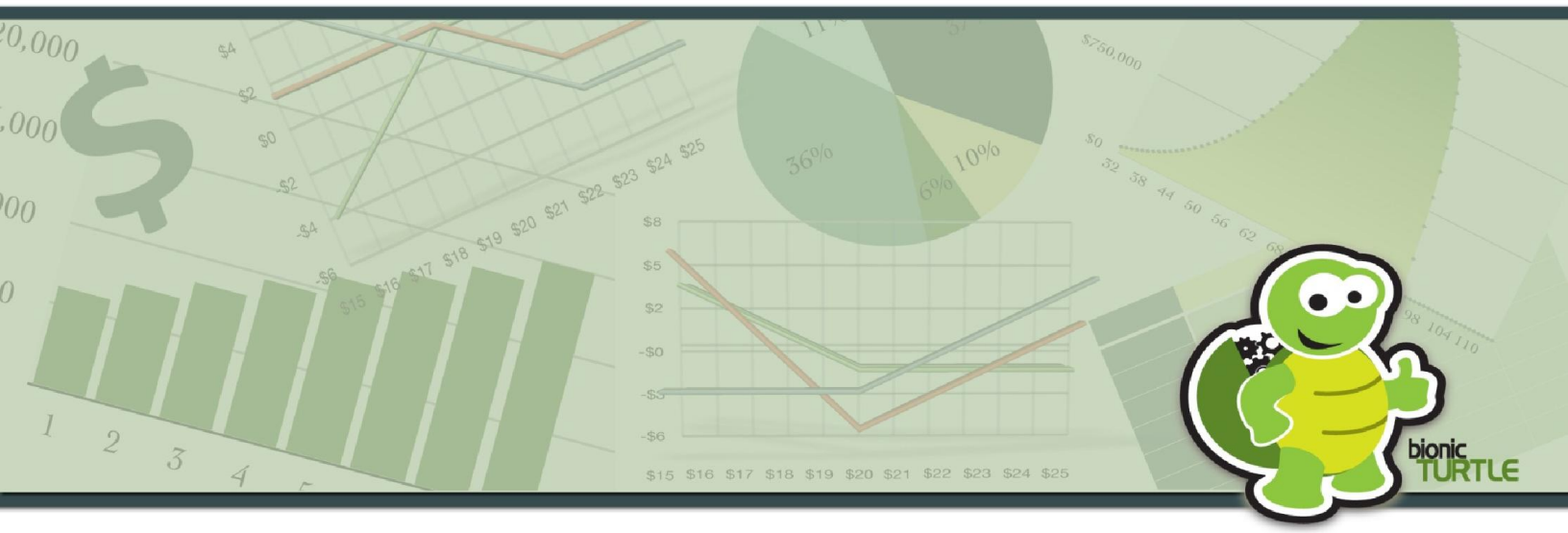

P1.T3. Financial Markets and Products

Chapter 20. Swaps

Bionic Turtle FRM Practice Questions

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Key Ideas According to David

Chapter 20. Swaps

Key Ideas According to David

Before 2020, these questions were covered by Hull, Options, Futures & Other Derivatives. Although GARP has separated the chapters into different readings, we are retaining these key ideas in each of the corresponding documents.

- Interest rates
- Futures/Forwards (Commodities)
- Interest rate futures
- Corporate Bonds
- Swaps
- Options and option trading strategies

Interest Rates

Three skills will put you in a good position: compound frequencies; present value (bond) pricing based on discounted cash flow; and implied forward rates given spot rates:

1. You **do need to be fluent with compound frequencies**. Probably, like the last exam, the default compound frequency will be annual. However, you still need to be ready to convert. If a rate is 8.0% per annum with annual compounding, you should easily be able to convert to its semi-annual and continuous equivalents.

2. Probably the most basic pricing skill is "vanilla" bond pricing; by vanilla, I refer to a basic coupon-bearing or zero-coupon bond. For example, given a zero rate curve (5% @ 0.5 years, 5.8% at 1.0 year, 6.4% at 1.5 years, and 6.8% at 2.0 years, each continuously compounding), what is the price of a two-year $100 face bond that pays a semi-annual coupon at a coupon rate of 6.0%. You should be able to do this.

3. ***GARP likes to test the implied forward rate given the spot rate curve***. You can almost expect to be asked. For example, if the 2-year spot rate is 1.2% and the 3-year spot rate is 1.4%, you should be able to infer the one-year forward rate, f(2,3), under continuous, annual, and/or semi-annual compound frequencies.
    - Please note that GARP like a price-based variation on the implied forward rate, which I reviewed here at http://www.bionicturtle.com/forum/threads/shortcut-to-forward-rates-if-you-have-bond-prices.4927/

Futures/Forwards (Commodities)

This is a potentially deep topic. But I think your focus should be on the following:

- Cost of carry; i.e., be facile with computing the implied model forward price. But please do not only practice solving for F(0). You must be sufficiently comfortable such that you can, for example, extract the convenience yield if given F(0).
- The minimum variance hedge ratio is extremely likely to be tested; I included two examples in the Focus Review, given it appears every year
- Please practice the optimal hedge (minimum variance) with both commodities and equity portfolios (hedged with index futures)
- I think margin accounts are testable (initial and maintenance margins for the futures positions that are used to hedge)
- With respect to futures contract sizes, I think you should know that T-bond futures are standardized at $100,000; Eurodollars at $1,000,000; and S&P 500 at a 250 multiple (*250). They are likely to be provided but are common enough that it helps to just know them. More detail here at http://www.bionicturtle.com/forum/threads/futures-contract-sizes.4959/
- Be comfortable with contango/backwardation (observed) and normal contango/backwardation (unobserved)

Do you need to memorize the size of commodity contracts?
Probably an exam question will provide you with contract size, rather than assume you know. Although, I do think it is good practice to know the following due to their exam popularity:

- Treasury bonds: $100,000 (GARP may assume you know)
- S&P 500: $250 * index futures price (popularly used for questions)
- Eurodollar: $1,000,000

And the following are not uncommon:

- Gold: 100 troy ounces (I agree with you)
- NASDAQ 100: $100 * index futures price
- S&P & NASDAQ MINI contracts: one-fifth (1/5th); i.e., $50* and $20*
- Crude oil: 1,000 barrels
- Silver: 5,000 ounces (maybe, do most people know this? I don't think so....)
- Corn (= wheat): 5,000 bushels (popular in quizzes)
- Copper: 25,000 pounds

Where can you find these? http://www.cmegroup.com/ e.g.,
http://www.cmegroup.com/trading/metals/base/copper_contract_specifications.html

Interest rate futures

I think you should focus on:

- Day count conventions
- Understanding the mechanics of the Eurodollar futures contract and Treasury bond futures contracts
- GARP likes to test cheapest-to-deliver (CTD); i.e., given three or four eligible bonds, identify the CTD
- Definitely be ready to compute the number of interest rate futures contracts used to duration hedge a fixed-income position. If you are given two durations, you do NOT want to hedge with the current durations, but RATHER the expected forward durations at maturity.

Swaps

I think the assignment (Hull) divides into three: comparative advantage; swap mechanics; and swap valuation. We have several practice questions on comparative advantage (it reduces to the observation that the total net gain equals the difference between fixed and floating rate differentials), but historically this tricky idea has barely been tested to my knowledge. You clearly need to be comfortable with swap mechanics so you can answer a very basic, non-quantitative question like one I included in the Focus Review (FR):

GARP 2010.P1.12. The yield curve is upward sloping, and a portfolio manager has a long position in 10-year Treasury Notes funded through overnight repurchase agreements. The risk manager is concerned with the risk that market rates may increase further and reduce the market value of the position. What hedge could be put on to reduce the position's exposure to rising rates?

a) Enter into a 10-year pay fixed and receive floating interest rate swap.
b) Enter into a 10-year receive fixed and pay floating interest rate swap.
c) Establish a long position in 10-year Treasury Note futures.
d) Buy a call option on 10-year Treasury Note futures.

Some key (exam) points to keep in mind with respect to swap mechanics:

- The vanilla interest rate swap (IRS) references notional; i.e., the notional is not exchanged (But the principal is exchanged in a currency swap; hence the maximum potential future [credit] exposure of a currency swap occurs at maturity)
- By default, the floating rate is determined at the beginning of each period and paid at the end; e.g., the first fixed-rate settlement is known at swap inception
- The duration of a swap position can be inferred from its valuation treatment as consisting of two bond legs: just as value[swap, POV of fixed-rate receiver, floating-rate payer] = value[fixed-rate bond] - value[floating-rate bond], the duration of the IRS from the perspective of the fixed-rate receiver (who is effectively long the fixed-rate bond-equivalent and short the floater) is approximately equal to the duration of the fixed-rate bond-equivalent. For example, the (modified) duration of a swap with a 3-year tenor, from the perspective of a 4.0% fixed rate payer is about 2.8 years at settlement because the duration equals 2.8 years (i.e., fixed-rate bond) minus about zero (duration of floating-rate bond is time-to-next-coupon).

In regard to swap valuation, you must practice a few. You'll quickly see that it's just like pricing a bond but with a tiny additional step, where the key insight is that the floating-rate bond-equivalent, for valuation purposes, only requires a single cash flow due to the elegant fact that it prices exactly at par at the next settlement. In the FR, I included the classic sort of swap valuation that you could see on the exam:

GARP 2011.P1.E1.10. A bank had entered into a 3-year interest rate swap for a notional amount of USD 300 million, paying a fixed rate of 7.5% per year and receiving LIBOR annually. Just after the payment was made at the end of the first year, the continuously compounded 1-year and 2-year annualized LIBOR rates were 7% per year and 8% per year, respectively. The value of the swap at that time was closest to which of the following choices?

Options and option trading strategies

In collecting the three-year sample of exam-type questions, I was surprised at the high prevalence of put-call parity in the FRM. Historically, put-call parity questions are very common. (Please note this is a T3 summary and does not include discussion of option pricing models, OPM, which are T4 topics). It is essential that you memorize, and are utterly comfortable with, the put-call parity formula; for example, can you, without any reference, quickly produce the formula's equivalent of a covered call or protective put?

After you have mastered the usage of the put-call parity, c + K*exp(-rT) = p + S, you might take a look at my method for dealing with an arbitrage exploitation question, see http://www.bionicturtle.com/forum/threads/how-to-work-put-call-parity-arbitrage-problems.6167/

Finally, I would be familiar with Hull's rules about the optimality of early exercise under the four permutations of call/put and European/American.

Option Trading Strategies

In my opinion, the section (a single Hull chapter) requires some of your time if you want to be fully ready. So far, it's always been included in the exam. And, as I mentioned in the FR audio, to illustrate how we lack a shortcut here, last year, GARP asked a question about box spreads, which totally surprised me as it's a really minor strategy. With respect to mechanics, Hull parses them into:

- Asset + option; e.g., protective put, covered call
- Spread strategies
- Combinations

While that is a fine way to grasp them, you are unlikely to encounter an exam question along these lines. Rather, you want to focus on applications and risk/reward perspective, with particular emphasis on upside/downside potential. For example,

- Which of the strategies is long volatility?
- Which of the strategies are directional;
  i.e., benefit from an increase/decrease in asset price?
- Which have capped or uncapped payouts?
- Which produces an initial cash inflow?

Chapter 20. Swaps

P1.T3.722. Using the swap rate to bootstrap the forward rate and basic interest rate swap valuation

Learning objectives: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows ... Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions. Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

722.1. Consider two firms, Reliable Corp and Dubious Corp. Reliable Corp has a strong balance sheet and repayment history; it can borrow 5.0% in fixed-rate loan markets or, in floating-rate markets, Reliable can pay 40 basis points above six-month LIBOR. Dubious Corp has a weaker balance sheet and must pay 7.50% in fixed-rate loan markets or, in floating-markets, Dubious must pay 310 basis points above six-month LIBOR.

Borrowing Rates in External Markets for Each Company

	Fixed Rate	Floating Rate
Reliable Corp	5.00%	6-month LIBOR + 0.40%
Dubious Corp	7.50%	6-month LIBOR + 3.10%

Which of the following statements is **TRUE** about this situation with respect to comparative advantage?

a) Neither has a comparative advantage in either market
b) Reliable has a comparative advantage in BOTH the fixed-rate and floating-rate markets
c) Reliable has a comparative advantage in floating-rate markets, but Dubious has a comparative advantage in fixed-rate markets
d) Reliable has a comparative advantage in fixed-rate markets, but Dubious has a comparative advantage in floating-rate markets

722.2. Suppose that some time ago, a financial institution entered into a swap where it agreed to make semi-annual payments at a rate of 3.0% per annum and receive LIBOR on a notional principal of $400.0 million. The swap now has a remaining life of only nine months (0.75 years). Payments will therefore be made 0.25 and 0.75 years from today. The risk-free rates with continuous compounding is assumed to be the LIBOR zero rate, and currently, it is 2.20% for all maturities. Because the LIBOR zero rate curve is flat at 2.20%, the six-month forward rate beginning in three months, F(0.25, 0.75), is also 2.20% with continuous compounding and therefore is equal to 2*[exp(0.0220/2)-1] = 2.2121% with semi-annual compounding. The LIBOR rate applicable to the exchange in 0.25 years was determined 0.25 years ago; suppose it was 3.0% with semi-annual compounding (LIBOR has dropped in the meantime). Note: the question was inspired by Hull's Example 7.1 in his 10th Edition.

Which is **nearest** to the present value of the swap to the financial institution?

a) -$1.550 million
b) -$287,300
c) +1.883 million
d) +2.940 million

722.3. Suppose that the 6-month, 12-month, 18-month, and 24-month overnight indexed swap (OIS) zero rates with continuous compounding are 2.00%, 2.40%, 3.00%, and 3.60%, respectively. Suppose further that the six-month LIBOR rate is 2.60% with semi-annual compounding. The forward LIBOR rate for the period between 6 and 12 months is 3.00%, with semi-annual compounding. The forward LIBOR rate for the period between 12 and 18 months is 3.60%, with semi-annual compounding. (Please Note: this question is inspired by Hull's Example 7.2 in 10th Edition).

Par	$100.00
2-year swap rate	4.00%

	OIS Zero	Forward	Cash Flow	
Period	rates	LIBOR	FV	PV
	(CC)	(s.a.)		
0.50	2.00%	2.600%	($0.700)	($0.693)
1.00	2.40%	3.000%	($0.500)	($0.488)
1.50	3.00%	3.600%	($0.200)	($0.191)
2.00	3.60%	???		

Finally and importantly, assume the two-year swap rate is 4.00%. Conditional on the realization of the LIBOR forward rates, the future cash flow in six months is, therefore (2.60% - 4.00%)/2 *$100.0 = -$0.70 and its present value is about -$0.70*exp(-0.020*0.50) = -$0.693; that is, we are using the OIS zero rates as the risk-free rate for discounting purposes.

Which is nearest to an estimate for the forward LIBOR rate for the 18- to 24-month period, F(1.5, 2.0)?

a) 3.880%
b) 4.503%
c) 5.747%
d) 6.950%

Answers:

722.1. C. TRUE: Reliable has a comparative advantage in floating-rate markets, but Dubious has a comparative advantage in fixed-rate markets.

Reliable has an ABSOLUTE advantage in both markets: it pays a lower rate in both markets. Reliable pays 2.50% less in the fixed-rate market but pays 2.70% less in the floating-rate market, where it has a comparative advantage. But Dubious pays 2.70% in the floating-rate market (where it has the comparative disadvantage) but pays only 2.50% more in the fixed-rate market, where it has the comparative advantage.

722.2. A. -$1.550 million.

We can value this swap as two bonds (see upper panel below). Although it might be easier to recognize that the exchange in three months will be zero, such that only the cash flow at nine months needs to be evaluated: The floating rate pays 0.5 * $400 million * 2*[exp(0.0220/2)-1] = $4.42429 million, and the fixed-rate pays $6.00; so the future (and final) net cash flow exchange is $1.57571, which has a present value of $1.54993 million.

BT 722.2. Vanilla swap valuation				
Assumptions				
Notional	**$400.00**			
Swap rate	**3.00%**			
Time	**-0.25**	**0.25**	**0.75**	
LIBOR	**3.00%**	**2.20%**	**2.20%**	
Discount Factor (CC)		0.9945	0.9836	
IRS value as two bonds (Hull Example 7.2)				
Floating Cash Flows				
Future value (FV)		$406.00		
Present value (PV)		$403.77		**$403.77**
Fixed Cash Flows				
Future value (FV)		$6.00	$406.00	
Present value (PV)		$5.97	$399.36	**$405.32**
				-$1.5499
IRS value as FRAs (Hull Example 7.3)				
Time		**0.25**	**0.75**	
LIBOR (continuous)		2.20%	2.20%	
Forward rates **(CC)**		2.20%	2.20%	
Forward rates **(s.a.)**		3.00%	2.21%	
Floating CFs (FV)		$6.00	$4.42	
Fixed CFs (FV)		$6.00	$6.00	
Net cash flows (FV)		$0.00	-$1.58	
Net cash flows (PV)		$0.00	-$1.55	**-$1.5499**

722.3. D. 6.950%. See below. The present value of the swap must be zero, such that the final cash flow must equal +1.372, and it's future value must be $1.372*exp(0.0360 * 2.0) = $1.475. Because 0.5 * [F(1.5, 2.0) - 4.0%] * $100 = $1.475 --> F(1.5, 2.0) = $1.475/($100*0.5) + 0.040 = 6.950%.

Par	$100.00
2 year swap rate	4.00%

Period (years)	OIS zero rates (CC)	Forward LIBOR (s.a.)	Cash Flow FV	Cash Flow PV	
0.50	2.00%	2.600%	($0.700)	($0.693)	
1.00	2.40%	3.000%	($0.500)	($0.488)	
1.50	3.00%	3.600%	($0.200)	($0.191)	
2.00	3.60%	6.950%	$1.475	$1.372	= $0.00 --$0.69 --$0.49 --$0.19
				$0.00	

6.950%	= $1.475/($100.00*0.5) + 4.00%

Discuss here in the forum: https://www.bionicturtle.com/forum/threads/p1-t3-722-using-the-swap-rate-to-bootstrap-the-forward-rate-and-basic-interest-rate-swap-valuation.10657/